

UDC: 621.311

DOI: <https://doi.org/10.30546/09085.2025.02.110.9078>

MODELING THE DEPENDENCE OF ARC OVERVOLTAGE ON GROUND FAULT RESISTANCE AND GROUND FAULT ANGLE

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ARTICLE INFO	ABSTRACT
Article history Received:2025-10-18 Received in revised form:2025-10-20 Accepted:2025-10-31 Available online Keywords: overvoltage multiplicity; ground fault resistance; ground fault angle; regression equation; correlation JEL classification: L60, L64, O33, C63	<i>In isolated neutral networks, the creation of artificial overvoltages is necessary for testing the insulation of electrical equipment under load conditions. In such scenarios, it becomes essential to establish the mathematical relationships between the parameters of single-phase non-stationary ground faults. During these faults, the dependencies between key parameters such as the multiplicity overvoltage, ground fault resistance and ground fault angle follow complex patterns. Therefore, it is important to develop adequate mathematical models that can describe the interdependencies of these parameters under practical conditions. This study addresses the analytical determination of the relationship between transient overvoltage, ground fault resistance, and fault angle in isolated neutral networks resulting from non-stationary ground faults. For this purpose, a regression equation was derived to express the dependence of the overvoltage multiplicity on ground fault resistance and fault angle, and the corresponding spatial representation was constructed. The obtained results confirm a strong correlation among these parameters, indicating their applicability for practical use.</i>

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1. Introduction

It is well known that the failure of electrical equipment can lead to various injuries among operational personnel, disruption of technological processes, and severe accidents. Therefore, specific tests are conducted in advance to prevent such issues. In general, these tests are carried out in the following cases: commissioning of equipment or installations, after an accident, following scheduled or unscheduled maintenance, or after a certain period since the last inspection. Notably, high-voltage insulation testing of electrical equipment is mandatory for isolated neutral power networks with voltages up to 35 kV [1, 2].

In isolated neutral networks, performing high-voltage insulation tests under load conditions is of particular relevance for identifying potential equipment failures in advance and ensuring the uninterrupted power supply to consumers [3, 4]. Research conducted in this field indicates that various methods and tools have been proposed for such testing [5–7].

In [8], a method for testing insulation under load in isolated neutral networks is proposed. According to this method, artificial transient ground faults are created in the network based on Petersen's theory in order to test the insulation under load conditions. It should be noted that during such ground faults, the magnitude of overvoltage caused by arcing in the network

depends on the ground fault resistance, the ground fault angle, and the phase-to-ground capacitance of the network. Therefore, determining the ground fault resistance and angle in advance is essential for accurately selecting the test voltage. For this purpose, it is a relevant and important task to determine the dependence of arc overvoltage multiplicity—arising from transient ground faults in isolated neutral networks—on ground fault resistance, ground fault angle, and phase-to-ground capacitance of the network.

2. Problem statement

In general, to determine the dependencies among the aforementioned parameters, it is necessary to perform a numerical solution of the system of differential equations characterizing the transient process of single-phase earth fault in neutral isolated networks, using modern computational technologies. However, the numerical solution of this problem is considerably complicated due to the stiffness of the mentioned differential equations. In other words, since the system of differential equations is nonlinear, in some cases the stability of the solution is compromised and the results become distorted. Therefore, to overcome such difficulties, it is essential to obtain analytical expressions defining the dependencies of the arc overvoltage multiplicity (k) on the earth fault resistance (R_0), the earth fault angle (φ), and the phase-to-earth capacitance of the network (C_f). It should be noted that for this purpose, the analytical dependencies of the arc overvoltage multiplicity on the earth fault resistance have been studied in [9,10], on the earth fault angle in [11,12], on the phase-to-earth capacitance in [13,14], and on both the earth fault resistance and the phase-to-earth capacitance in [15]. Continuing these studies, this work considers the derivation of a regression model describing the dependence of single-phase arc overvoltage in neutral isolated networks on the earth fault resistance and earth fault angle.

3. Problem solving

The derivation of an analytical expression for the dependency of the arc overvoltage multiple, occurring during single-phase faults in an isolated neutral electrical network ($C_f = \text{const}$), on the earth fault resistance and earth fault angle is considered. For this purpose, the results of experimental investigations conducted on a low-voltage model of an isolated neutral network ($C_f = 1\text{mkF}$) are utilized, as presented in Table 1 [8].

Table 1.: Dependence $k = f(R_0, \varphi)$

R_0, Ohm	φ				
	30°	60°	90°	120°	150°
5	2,45	3,16	3,30	3,10	2,16
10	2,29	2,91	2,96	2,86	2,03
15	2,16	2,69	2,77	2,66	1,93
20	2,05	2,51	2,62	2,49	1,84
25	1,96	2,35	2,49	2,35	1,76
30	1,88	2,22	2,38	2,24	1,69

As seen from Table 1, the dependence between the multiple of the arc overvoltage, the ground fault resistance, and the ground fault angle can be approximated by the following regression equation [16]:

$$k = \frac{a}{R_0} + b \sin \varphi + c, \quad (1)$$

here, a, b, c – are regression coefficients.

If we introduce the substitutions $\frac{1}{R_0} = x$ and $\sin \varphi = y$ in equation (1), the regression equation can be written in the following form:

$$k = ax + by + c, \quad (2)$$

In other words, the dependence of the arc overvoltage multiplicity (k) on the conductivity of the ground fault circuit (x) and the sine of the ground fault angle (y) can be approximated by a linear regression equation (see Table 2). The choice of this model type is based on the assumption that if the variation of the output variable is directly proportional to the variations of the factor variables, then a linear model is considered adequate [17].

Table 2.: Dependence $k = f(x, y)$

x [Sm]	y				
	0,500	0,866	1,000	0,866	0,500
0,200	2,45	3,16	3,30	3,10	2,16
0,100	2,29	2,91	2,96	2,86	2,03
0,067	2,16	2,69	2,77	2,66	1,93
0,050	2,05	2,51	2,62	2,49	1,84
0,040	1,96	2,35	2,49	2,35	1,76
0,033	1,88	2,22	2,38	2,24	1,69

The regression coefficients of equation (2) are defined by the following well-known expressions [17]:

$$\left. \begin{aligned} a &= \frac{\sigma(k)}{\sigma(x)} \cdot \frac{r(k, x) - r(k, y)r(x, y)}{1 - r^2(x, y)}; \\ b &= \frac{\sigma(k)}{\sigma(y)} \cdot \frac{r(k, y) - r(k, x)r(x, y)}{1 - r^2(x, y)}; \\ c &= M(k) - aM(x) - bM(y). \end{aligned} \right\}, \quad (3)$$

where $M(x)$, $M(y)$ and $M(k)$ denote the mathematical expectations (means) of the variables x , y , and k , respectively; $\sigma(x)$, $\sigma(y)$, and $\sigma(k)$ denote the standard deviations of the variables x , y , and k , respectively; $r(x, y)$ is the linear correlation coefficient between x and y ; $r(k, x)$ is the linear correlation coefficient between k and x ; and $r(k, y)$ is the linear correlation coefficient between k and y .

The numerical values of the necessary statistical indicators for determining the regression coefficients and identifying the target model are calculated based on the correlation matrix presented in Table 3. The following notation is used for the correlation matrix:

$$\begin{aligned} A_i &= (x_i - M(x))^2; \quad B_i = (y_i - M(y))^2; \quad C_i = (k_i - M(k))^2; \\ E_i &= (x_i - M(x)) \cdot (y_i - M(y)); \quad P_i = (k_i - M(k)) \cdot (x_i - M(x)); \quad T_i = (k_i - M(k)) \cdot (y_i - M(y)); \\ L_i &= \left| \frac{ax_i + by_i + c - k_i}{k_i} \right|; \quad N_i = (ax_i + by_i + c - k_i)^2. \end{aligned}$$

Table 3.: Correlation Table

i	x_i	y_i	k_i	A_i	B_i	C_i	E_i	P_i	T_i	L_i	N_i
1	0,200	0,500	2,45	0,01400278	0,06071797	0,001708	-0,02915854	0,0048911	-0,0101850	0,028818943	0,004985265
2	0,200	0,866	3,16	0,01400278	0,01430781	0,564502	0,01415447	0,0889078	0,0898709	0,022324562	0,004976684
3	0,200	1,000	3,30	0,01400278	0,06430781	0,794475	0,03000813	0,1054744	0,2260331	0,000706912	5,44199E-06
4	0,200	0,866	3,10	0,01400278	0,01430781	0,477942	0,01415447	0,0818078	0,0826940	0,003401812	0,00011121
5	0,200	0,500	2,16	0,01400278	0,06071797	0,061835	-0,02915854	-0,0294256	0,0612740	0,166947412	0,130036983
6	0,100	0,500	2,29	0,00033611	0,06071797	0,014082	-0,00451752	-0,0021756	0,0292407	0,08192638	0,035198042
7	0,100	0,866	2,91	0,00033611	0,01430781	0,251335	0,00219295	0,0091911	0,0599671	0,082049292	0,05700798
8	0,100	1,000	2,96	0,00033611	0,06430781	0,303968	0,00464915	0,0101078	0,1398125	0,027213051	0,006488404
9	0,100	0,866	2,86	0,00033611	0,01430781	0,203702	0,00219295	0,0082744	0,0539863	0,066001202	0,035631636
10	0,100	0,500	2,03	0,00033611	0,06071797	0,143388	-0,00451752	-0,0069422	0,0933073	0,035659403	0,005240108
11	0,067	0,500	2,16	0,00022500	0,06071797	0,061835	0,00369615	0,0037300	0,0612740	0,091211737	0,038815837
12	0,067	0,866	2,69	0,00022500	0,01430781	0,079148	-0,00179423	-0,0042200	0,0336518	0,058799026	0,025017553
13	0,067	1,000	2,77	0,00022500	0,06430781	0,130562	-0,00380385	-0,0054200	0,0916305	0,010814646	0,000897396
14	0,067	0,866	2,66	0,00022500	0,01430781	0,063168	-0,00179423	-0,0037700	0,0300633	0,048183977	0,01642739
15	0,067	0,500	1,93	0,00022500	0,06071797	0,229122	0,00369615	0,0071800	0,1179483	0,017089455	0,001087855
16	0,050	0,500	2,05	0,00100278	0,06071797	0,128642	0,00780299	0,0113578	0,0883791	0,076448938	0,024561259
17	0,050	0,866	2,51	0,00100278	0,01430781	0,010268	-0,00378782	-0,0032089	0,0121210	0,019072649	0,002291762
18	0,050	1,000	2,62	0,00100278	0,06430781	0,044662	-0,00803034	-0,0066922	0,0535920	0,019213916	0,002534162
19	0,050	0,866	2,49	0,00100278	0,01430781	0,006615	-0,00378782	-0,0025756	0,0097287	0,011193715	0,000776868
20	0,050	0,500	1,84	0,00100278	0,06071797	0,323382	0,00780299	0,0180078	0,1401252	0,028956347	0,002838724
21	0,040	0,500	1,96	0,00173611	0,06071797	0,201302	0,01026709	0,0186944	0,1105560	0,055378625	0,011781388
22	0,040	0,866	2,35	0,00173611	0,01430781	0,003442	-0,00498397	0,0024444	-0,0070174	0,029917391	0,004942915
23	0,040	1,000	2,49	0,00173611	0,06430781	0,006615	-0,01056624	-0,0033889	0,0206253	0,055629991	0,019187424
24	0,040	0,866	2,35	0,00173611	0,01430781	0,003442	-0,00498397	0,0024444	-0,0070174	0,029917391	0,004942915
25	0,040	0,500	1,76	0,00173611	0,06071797	0,420768	0,01026709	0,0270278	0,1598381	0,051964713	0,008364547
26	0,033	0,500	1,88	0,00233611	0,06071797	0,279488	0,01190982	0,0255522	0,1302688	0,03001239	0,003183588
27	0,033	0,866	2,22	0,00233611	0,01430781	0,035595	-0,00578140	0,0091189	-0,0225674	0,077668775	0,02973027
28	0,033	1,000	2,38	0,00233611	0,06430781	0,000822	-0,01225684	0,0013856	-0,0072696	0,092704827	0,048680901
29	0,033	0,866	2,24	0,00233611	0,01430781	0,028448	-0,00578140	0,0081522	-0,0201751	0,068046732	0,023233283
30	0,033	0,500	1,69	0,00233611	0,06071797	0,516482	0,01190982	0,0347356	0,1770868	0,079039472	0,017842737
*	2,450	22,392	72,26	0,09819444	1,28615612	5,390747	0,00000000	0,4106667	1,998843	1,4663	0,5668

The data array consists of a volume of $n=30$, and the values of the statistical indicators necessary for determining the coefficients, obtained as a result of calculations, are presented below.

$$\sum_{i=1}^n x_i = 2,45; \sum_{i=1}^n y_i = 22,392; \sum_{i=1}^n k_i = 72,26;$$

$$M(x) = \frac{\sum_{i=1}^n x_i}{n} = 0,082; M(y) = \frac{\sum_{i=1}^n y_i}{n} = 0,746; M(k) = \frac{\sum_{i=1}^n k_i}{n} = 2,409;$$

$$\sum_{i=1}^n (x_i - M(x))^2 = 0,09819444; \sum_{i=1}^n (y_i - M(y))^2 = 1,28615612; \sum_{i=1}^n (k_i - M(k))^2 = 5,390747;$$

$$\sum_{i=1}^n (x_i - M(x))(y_i - M(y)) = 0; \sum_{i=1}^n (k_i - M(k))(x_i - M(x)) = 0,4106667;$$

$$\sum_{i=1}^n (k_i - M(k))(y_i - M(y)) = 1,998843;$$

$$\sum_{i=1}^n \left| \frac{ax_i + by_i + c - k_i}{k_i} \right| = 1,4663;$$

$$\sum_{i=1}^n (ax_i + by_i + c - k_i)^2 = 0,5668.$$

Based on Table 3, variances, root mean square deviations, two-dimensional covariance coefficients (correlation moments), and two-dimensional correlation coefficients for individual quantities are calculated using well-known formulas. The obtained numerical values are as follows:

$$D(x) = \frac{\sum_{i=1}^n (x_i - M(x))^2}{n} = 0,00327; D(y) = \frac{\sum_{i=1}^n (y_i - M(y))^2}{n} = 0,04287;$$

$$D(k) = \frac{\sum_{i=1}^n (k_i - M(k))^2}{n} = 0,17969;$$

$$\sigma(x) = \sqrt{D(x)} = 0,0572; \sigma(y) = \sqrt{D(y)} = 0,2071; \sigma(k) = \sqrt{D(k)} = 0,4239;$$

$$\text{cov}(x, y) = \frac{\sum_{i=1}^n (x_i - M(x))(y_i - M(y))}{n} = 0; \text{cov}(k, x) = \frac{\sum_{i=1}^n (k_i - M(k))(x_i - M(x))}{n} = 0,01369;$$

$$\text{cov}(k, y) = \frac{\sum_{i=1}^n (k_i - M(k))(y_i - M(y))}{n} = 0,06663;$$

$$r(x, y) = \frac{\text{cov}(x, y)}{\sqrt{D(x)D(y)}} = 0; r(k, x) = \frac{\text{cov}(k, x)}{\sqrt{D(k)D(x)}} = 0,56444; r(k, y) = \frac{\text{cov}(k, y)}{\sqrt{D(k)D(y)}} = 0,75911.$$

Then, based on expressions (3), the estimated values of the regression coefficients of equation (1) or (2) are obtained as follows:

$$a = 4,18; b = 1,55; c = 0,91.$$

Thus, once the regression coefficients have been determined, the relationship (2) between the multiplicity of the arc overvoltage arising during single-phase non-stationary ground faults in isolated neutral networks, the conductivity of the ground fault loop, and the sine of the ground fault angle can be explicitly expressed as follows:

$$k = 4,18x + 1,55y + 0,91, (4)$$

Let us verify the adequacy of the obtained regression dependence (4) between the conductivity of the single-phase transient ground fault circuit and the sine of the ground fault angle in relation to the overvoltage multiplicity during single-phase non-stationary ground faults. To do this, we can calculate the multiple correlation coefficient and assess its significance using the Fisher criterion [17].

The value of the multiple correlation coefficient is determined by the following well-known expression:

$$R = \sqrt{\frac{r^2(k, x) + r^2(k, y) - 2r(k, x)r(k, y)r(x, y)}{1 - r^2(x, y)}} = 0,95.$$

A multiple correlation coefficient close to one ($R = 0,95 \rightarrow 1$) indicates that the relationship between the recurrence of arc overvoltages, the conductivity of the ground fault circuit, and the sine of the ground fault angle can be considered a strong linear correlation.

The significance of the multiple correlation coefficient is tested using the F-Fisher criterion. It is known that, at a significance level of α , the regression equation is considered adequate if the condition $F > F(\alpha, q_1, q_2)$ is satisfied [17], where q_1 and q_2 are the degrees of freedom.

The empirical value of the F-Fisher criterion, based on the given data, is determined as follows:

$$F = \frac{R^2}{1-R^2} \cdot \frac{n-m-1}{m}, \quad (5)$$

Here, n is the number of experiments, $n = 30$; m is the number of factors, $m = 2$. According to equation (5), the value of $F = 125,4$.

The critical value of the Fisher criterion F is obtained from the table depending on the significance level (α) and the degrees of freedom (q_1, q_2) [17]:

$$\alpha = 0,05; \quad q_1 = m = 2; \quad q_2 = n - m - 1 = 30 - 2 - 1 = 27; \quad F(\alpha, q_1, q_2) = 3,35.$$

Since $F = 125,4 > F(\alpha, q_1, q_2) = 3,35$, the multivariate correlation coefficient ($R = 0,95$) and the statistical significance of the regression equation are confirmed.

The mean relative error and the mean square error of the approximation are determined by the following known expressions, respectively:

$$\bar{\varepsilon} = \frac{1}{n} \sum_{i=1}^n \left| \frac{ax_i + by_i + c - k_i}{k_i} \right| \cdot 100\% = 4,89\%;$$

$$\sigma = \sqrt{\frac{\sum_{i=1}^n (ax_i + by_i + c - k_i)^2}{n}} = 0,137.$$

The determination coefficient of $R^2 = 0,95^2 = 0,9025$ indicates that 90,25% of the variation in the arc overvoltage multiplication factor (k) is primarily caused by changes in the ground fault circuit conductance (x) and the sine of the ground fault angle (y), while the remaining variation 9,75 % is attributed to other unaccounted factors.

Thus, the dependence of the arc overvoltage multiplication factor on the ground fault resistance and the ground fault angle in neutral isolated networks under non-stationary ground faults can be explicitly expressed as follows:

$$k = \frac{4,18}{R_0} + 1,55 \sin \varphi + 0,91, \quad (6)$$

As can be seen, during single-phase transient ground faults, the regression model obtained in the form of equation (6) represents a simple and practically applicable relationship between the parameters.

Based on the regression equation derived using the OriginLab software [18], a 3D (spatial) representation of the dependency of the arc overvoltage multiplication factor on the ground fault resistance and the ground fault angle has been constructed (Figure 1).

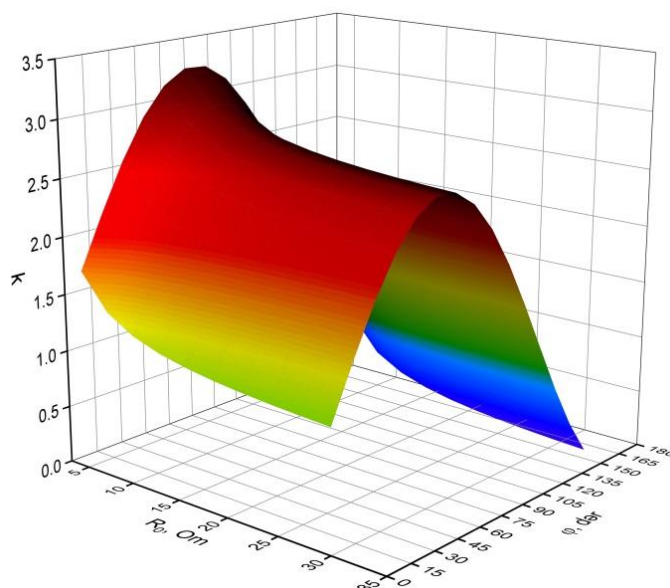


Figure 1. 3D visualization of the dependence of arc overvoltage multiplicity on ground fault resistance and ground fault angle

4. Conclusion

1. A practically implementable regression model has been developed to describe the dependence between the arc overvoltage multiplicity, ground fault resistance, and ground fault angle in neutral-isolated networks subjected to non-stationary ground faults governed by the Petersen theory.
2. The obtained regression model can be readily applied for insulation testing under load conditions in neutral-isolated networks of the Azerenergy system, as well as for the investigation and analysis of results related to non-stationary ground faults occurring in the network.

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