UDC: 004
DOI: https://doi.org/10.30546/09085.2025.02.313

ANALYSIS OF MIGRAINE AURA BASED ON EEG TESTS USING ARTIFICIAL INTELLIGENCE METHODS

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ARTICLE INFO

Article history
Received:2025-10-01
Received in revised form:2025-10-05
Accepted:2025-10-15
Available online

Keywords:
EEG Tests;
fully connected neural networks; artificial intelligence;
FitzHugh-Nagumo model;
Milstein method.

JEL classification: I10,I12,C45,C55

ABSTRACT

The dynamics of the membrane potential and recovery mechanism of biological neurons during migraine attacks with aura were mathematically modeled using the FitzHugh-Nagumo nonlinear stochastic differential equation system. The degree of influence of the membrane potential on the recovery mechanism, the degree of self-regulation of the recovery mechanism, and the stochastic resonance intensity coefficients affecting both components were determined using a fully connected neural network. This study addresses a significant gap in computational neuroscience by integrating stochastic differential equations with machine learning to characterize neuronal behavior during pathological conditions. Traditional electroencephalography (EEG) analysis methods primarily rely on time-frequency decomposition and statistical techniques, identifying fundamental signal characteristics but remaining disconnected from mechanistic stochastic neuronal models. Our approach combines the biophysically realistic FitzHugh-Nagumo framework with neural network-based parameter estimation using EEG recordings, enabling precise quantification of key biophysical parameters governing neuronal excitability and recovery processes. The identified parameters provide quantitative measures of membrane dynamics and stochastic fluctuations characterizing migraine pathophysiology, offering potential biomarkers for clinical diagnosis and personalized treatment strategies.

1. Introduction

Electroencephalography (EEG) tests are essential neurological diagnostic methods used to record and analyze brain wave activity (Niedermeyer & da Silva, 2004). EEG plays an important role in diagnosing various neurological conditions by recording the synchronized activity of cortical neurons, particularly valuable for studying rapid neurophysiological changes in disorders such as migraine with aura (Wolpaw et al., 2002).

EEG signals consist of waves with varying amplitudes and frequencies, divided into five types: delta (0.5-4 Hz), theta (4-8 Hz), alpha (8-13 Hz), beta (13-30 Hz), and gamma (>30 Hz) (Steriade et al., 1993). Each wave type is associated with specific cognitive and physiological states and reflects distinct patterns of neuronal synchronization across cortical networks (Buzsáki & Draguhn, 2004). Delta waves occur during deep sleep and are characterized by high amplitude and low frequency, representing widespread synchronization of cortical neurons.

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Theta waves relate to emotional states (Deco & Jirsa, 2012), memory encoding, and creative processes, particularly prominent in the hippocampus and temporal regions.Alpha waves predominate in relaxed but awake states (Klimesch, 1999), typically observed when individuals are at rest with eyes closed, and are most prominent in posterior brain regions. Beta waves are associated with active thinking, focus, and motor activity, reflecting desynchronized cortical states during cognitive engagement. Gamma waves represent the highest frequency oscillations and are linked to sensory processing, attention, and consciousness, often considered markers of neural integration across distributed brain networks (Buzsáki, 2006). Understanding these patterns is crucial for identifying abnormal neuronal activities during migraine aura episodes, as pathological states often manifest as alterations in the power, coherence, and spatial distribution of these frequency bands. Migraine with aura involves cortical spreading depression (CSD), characterized by neuronal depolarization waves followed by activity suppression propagating across the cortex at approximately 2-5 mm/min, which produces characteristic changes in EEG patterns during aura phases.The FitzHugh-Nagumo model (FitzHugh, 1961; Nagumo et al., 1962), a simplified version of the Hodgkin-Huxley model, effectively describes neuronal dynamics through differential equations. Its stochastic extension captures inherent biological randomness, making it well-suited for modeling pathological conditions like migraine aura (Gardiner, 2004; Wiesenfeld & Moss, 1995). Despite advances in mathematical neuroscience, existing approaches have limitations. Izhikevich (2007) contributed significantly to modeling neuronal dynamics but did not deeply address stochastic effects and parametric analysis of EEG data. Traditional EEG analysis methods using time-frequency techniques (Delorme & Makeig, 2004; Lotte et al., 2007) revealed signal characteristics but lacked integration with stochastic neuronal models. Deco and colleagues (Deco & Jirsa, 2012; Deco et al., 2008) studied stochastic processes in neural models but did not apply artificial neural networks for systematic parameter estimation from clinical data.

This work addresses these gaps by integrating machine learning with stochastic differential equations. We employ a multi-layer perceptron (MLP) neural network with ADAM optimization (Kingma & Ba, 2015) to estimate five key biophysical parameters from EEG data: internal self-regulation (a), membrane potential influence (b), neighbor influence (c), and stochastic resonance intensities (σ_R, σ_P). Using 1230 EEG recordings from migraine patients, we validate our approach and solve the stochastic system using the Milstein numerical method.

This study contributes to clinical neuroscience by providing quantitative biomarkers for migraine aura, enabling early detection and personalized treatment strategies. The paper is organized as follows: Section 2 reviews related work, Section 3 presents the stochastic FitzHugh-Nagumo model, Section 4 describes the MLP parameter estimation methodology, Section 5 shows visualization results, Section 6 details the numerical solution, Section 7 presents experimental findings, and Section 8 concludes with future directions.

2. Related works

The intersection of EEG analysis, mathematical modeling of neuronal dynamics, and artificial intelligence methods has been the subject of extensive research in recent years. This section reviews key contributions in these domains and positions the current study within the broader research landscape. The Hodgkin-Huxley model established the foundation for mathematical neuroscience by providing a detailed biophysical description of action potential generation. However, its computational complexity motivated the development of simplified models.

FitzHugh (1961) and Nagumo et al. (1962) independently developed a two-variable model that captures the essential excitable dynamics of neurons while maintaining computational tractability. This model has been extensively used to study various neuronal phenomena, including oscillations, bursting, and synchronization (Izhikevich, 2007). Stochastic extensions of the FitzHugh-Nagumo model have been developed to account for inherent randomness in biological systems. Wiesenfeld and Moss (1995) investigated the role of noise in neuronal dynamics and demonstrated that stochastic fluctuations can enhance signal detection through stochastic resonance. Glass (2001) further explored how noise affects the dynamics of excitable systems and their response to periodic stimuli. These studies established the theoretical foundation for incorporating stochastic processes into neuronal models but did not address parameter estimation from experimental data. Traditional EEG analysis has relied heavily on spectral methods and statistical approaches. Niedermeyer and da Silva (2004) provided comprehensive coverage of EEG interpretation in clinical settings, establishing standards for identifying pathological patterns. Buzsáki (2006) and Buzsáki and Draguhn (2004) explored the rhythmic nature of brain activity and its relationship to cognitive processes, emphasizing the importance of oscillatory dynamics in neural computation. Klimesch (1999) specifically analyzed how alpha and theta oscillations reflect cognitive and memory performance.

For migraine research, several studies have characterized EEG patterns during and between attacks, noting alterations in various frequency bands. Machine learning methods have been applied to classify migraine EEG patterns, demonstrating the potential of artificial intelligence in clinical diagnosis (Lotte et al., 2007). However, these studies focused primarily on pattern recognition rather than underlying biophysical mechanisms. The phenomenon of cortical spreading depression (CSD) is widely recognized as the neurophysiological correlate of migraine aura. Mathematical models of CSD propagation have demonstrated how reaction-diffusion equations can capture the spatial dynamics of spreading depression waves. These studies have provided detailed mechanistic insights into ionic mechanisms and neurotransmitter roles in CSD initiation and propagation but have not integrated stochastic modeling with machine learning approaches for parameter estimation from clinical data.

The application of artificial intelligence methods to neuroscience has expanded rapidly. LeCun et al. (2015) and Goodfellow et al. (2016) established deep learning foundations that have been applied to various neuroscience problems. Delorme and Makeig (2004) developed EEGLAB, an open-source toolbox for EEG analysis that has become widely used. Lotte et al. (2007) reviewed classification algorithms for EEG-based brain-computer interfaces, while Schmidhuber (2015) provided a comprehensive overview of deep learning in neural networks. Neural networks have been employed for parameter estimation in dynamical systems, with methods capable of discovering governing equations and estimating parameters from noisy data. The ADAM optimization method (Kingma & Ba, 2015) has proven particularly effective for training deep neural networks. However, these methods have not been specifically applied to estimate biophysical parameters of stochastic neuronal models from EEG data in migraine patients.

While previous studies have made significant contributions in mathematical modeling of neurons, EEG analysis, and AI methods separately, there remains a gap in integrating these approaches for studying migraine aura. Existing mathematical models often lack direct connection to clinical EEG data, while EEG analysis methods typically do not incorporate biophysically realistic stochastic models. Furthermore, although machine learning has been applied to EEG classification (Cortes & Vapnik, 1995; Duda et al., 2001), its use for estimating

specific biophysical parameters of neuronal dynamics remains underexplored. This study addresses these gaps by: (1) employing a stochastic FitzHugh-Nagumo model that captures both deterministic dynamics and random fluctuations relevant to migraine aura, (2) using a fully connected multi-layer perceptron neural network to estimate five key biophysical parameters $(a, b, c, \sigma_R, \sigma_P)$ directly from EEG band power recordings, (3) applying the Milstein numerical method (Kloeden & Platen, 1992) for accurate solution of the stochastic differential equation system, and (4) validating the approach on a comprehensive dataset of 1230 EEG recordings from migraine patients with aura.

3. Mathematical model of the migrain attack process

The FitzHugh-Nagumo model plays an important role in the scientific and research field of modeling neurological biomedical processes, measuring the dynamics of biological neurons, and understanding the excitation and recovery patterns in neural tissues. This model, which is a simplified version of the Hodgkin-Huxley model, describes the electrode impulses occurring in biological neurons as a system of differential equations (FitzHugh, 1961). The excitation, membrane potential and recovery mechanisms of biological neurons are modeled together and, as a result, are explained by complex processes such as the interaction of biological neurons, the generation and transmission of impulses. The stochastic model is widely used to study not only the specific behavior of specific neurons, but also the general dynamics in biological neural networks. The synchronization rules of electrode-based impulses in neural networks, that is, the parallel electrode-based impulse transmission of neurons, play an important role in various brain functions such as attention, memory and sensory processing (Glass, 2001). This system also explains the stopping or delay processes in the patient's neural networks during pathological conditions such as epilepsy. During epileptic seizures, excessive synchronization of neurons leads to abnormal impulse activity. The mathematical elegance of the FitzHugh-Nagumo model lies in its reduction of the four-dimensional Hodgkin-Huxley system to a two-dimensional framework, capturing the essential nonlinear dynamics of neuronal excitability while significantly reducing computational complexity (Nagumo et al., 1962). This dimensionality reduction is achieved by grouping fast variables related to membrane potential dynamics and slow variables associated with recovery processes, enabling efficient simulation of large-scale neural networks. The model exhibits rich dynamical behavior including stable fixed points, limit cycles, and bifurcations that correspond to different physiological states of neurons such as resting, oscillatory, and excitable regimes (Izhikevich, 2007). One of the key advantages of the FitzHugh-Nagumo framework is its ability to reproduce qualitatively similar dynamics to more complex biophysical models while maintaining analytical tractability for studying phase transitions and stability properties.

The incorporation of stochastic components into the FitzHugh-Nagumo model is essential for capturing the inherent variability observed in biological systems, arising from sources such as random channel openings, synaptic noise, and fluctuations in ion concentrations (Gardiner, 2004). Stochastic resonance, a phenomenon where optimal levels of noise enhance signal detection and transmission, has been extensively studied using stochastic versions of the FitzHugh-Nagumo model (Wiesenfeld & Moss, 1995). This phenomenon is particularly relevant in understanding how biological neurons can reliably transmit information despite noisy cellular environments and has implications for understanding sensory processing and neural coding. In the context of migraine with aura, the stochastic FitzHugh-Nagumo model provides a natural

framework for representing the complex interplay between deterministic cortical spreading depression dynamics and random fluctuations that influence the initiation, propagation, and termination of aura episodes.

Furthermore, the FitzHugh-Nagumo model has been successfully applied to simulate various pathological neural conditions beyond epilepsy, including Parkinson's disease tremors, cardiac arrhythmias, and sleep disorders, demonstrating its versatility as a computational tool in biomedical research (Deco et al., 2008). The model's parameters can be interpreted in terms of biophysical quantities such as membrane capacitance, ionic conductances, and recovery time constants, providing a direct link between mathematical abstractions and physiological mechanisms. Recent advances in numerical methods for stochastic differential equations, particularly the Milstein method (Kloeden & Platen, 1992), have enabled accurate simulation of stochastic FitzHugh-Nagumo systems with proper treatment of both drift and diffusion terms, ensuring reliable reproduction of statistical properties observed in experimental recordings.

Therefore, we will use a nonlinear stochastic FitzHugh-Nagumo differential equation system to model the electrodical activity of neurons in the patient's brain during migraine attacks [Eq. 1].

Thus,

$$\begin{cases} dP = \left(P - \frac{P^3}{3} - R\right) dt + \sigma_P \, dR_P \\ dR = (a + b \cdot P - c \cdot R)dt + \sigma_R \, dR_R \end{cases} \qquad (1)$$

For this equation;

$P[t]: [0, T] \to \mathbb{R}$ - is an upper bounded function defined in real Euclidean space that determines the stochastic dynamics of the membrane potential of biological neurons varying with time.

$R[t]: [0, T] \to \in \mathbb{R}$ - determines the dynamics of the stochastic recovery mechanism of a biological neuron.

$a \in \mathbb{R}$ - is the positive internal self-regulation parameter of a biological neuron.

$b \in \mathbb{R}$ - is a parameter describing the degree of influence of the membrane potential on the stochastic recovery mechanism.

$c \in \mathbb{R}$ - is a parameter describing the degree of influence of neighboring neurons on the recovery mechanism [22].

$\sigma_R \, dR_R$ - is a Wiener process, defined by a Gaussian distribution in real Euclidean space, and describes the stochastic resonance affecting the recovery mechanism [2], [27].

$\sigma_p \, dR_p$ - is a Wiener process defined by a Gaussian distribution in real Euclidean space, describing the stochastic resonance affecting the dynamics of the membrane potential [5], [10], [27].

In this section, we have established the mathematical foundation for modeling migraine aura dynamics using the stochastic FitzHugh-Nagumo differential equation system. The proposed model captures both the deterministic aspects of neuronal membrane potential and recovery mechanisms, as well as the stochastic fluctuations inherent in biological systems through Wiener processes. The five key parameters $(a, b, c, \sigma_R, \sigma_P)$ in the system represent biophysically meaningful quantities: internal self-regulation, membrane potential influence on recovery, neighbor neuron interactions, and stochastic resonance intensities affecting both membrane dynamics and recovery processes. This formulation provides a rigorous mathematical

framework that connects theoretical neuronal dynamics with observable EEG patterns during migraine attacks with aura. The challenge now lies in estimating these parameters from experimental EEG data, which requires sophisticated machine learning techniques capable of handling the nonlinear and stochastic nature of the system. In the following section, we address this challenge by developing a multi-layer perceptron neural network architecture specifically designed for parameter estimation from EEG frequency band features.

4. Parameter setting with multi-layer perceptron regressor

When solving a system of stochastic differential equations, since there are often no obvious analytical forms of the functions sought, approximate solutions of such systems are found by numerical methods. In the deterministic part of the system of equations under consideration, we were able to describe the dynamics of neurons with nonlinear components [14]. It is known that the stochastic part of the system (dR_R, dR_P) describes the Wiener process, and the Milstein method expresses this nonlinear process more precisely.

Before considering the numerical solution of the differential equation system considered in [Eq. 1] using the Milstein method, let us build a multi-layer perceptron regressor model with a fully connected neural network to determine the values of the coefficients a, b, c in the deterministic part of the system and σ_R, σ_P in the stochastic part. For this, based on 1230 EEG tests of patients with migraine with aura taken from the databases [7], [18], we obtain a five-dimensional dataset as a result of eight-dimensional dataset preprocessing that reflecting the average mutual electrode conduction amplitudes between the *frontal (F), frontopolar (Fp), temporal (T), central (C), parietal (P)* and *occipital (O)* lobes of the brain. The feature vector [Eq. 2] of this dataset contains *delta, theta, alpha, beta* and *gamma* brain waves [1] as attributes.

$$X = [\delta, \theta, \alpha, \beta, \gamma\ ] \qquad (2)$$

In the output layer of the constructed neural network [25], we can see a 5-dimensional row matrix [Eq. 3] that contains the parameters in the system [Eq. 1].

$$Y = [a, b, c, \sigma_R, \sigma_P] \quad (3)$$

Also, based on the medical-mathematical studies conducted [15], we can give the following [Eq. 4] rule for the parameters that we will estimate with the multi-layer perceptron regressor.

$$Y_{true} = \left[X_1 + X_2 - X_3, X_4 + X_5, X_3 + X_5, \frac{1}{X_1+X_2}, \frac{1}{X_4+X_5}\right] \qquad (4)$$

For the constructed neural network, the ReLU activation function is used, taking into account the nonlinearity of the system [Eq. 1] For a fully connected neural network [12] consisting of two hidden layers, the results for each hidden layer are found using the following [Eq. 5 – Eq. 6] formulas.

$$H_1 = ReLU(W_1 \cdot X + b_1) = \max(0, W_1 \cdot X + b_1) \quad (5)$$

$$H_2 = ReLU(W_2 \cdot H_1 + b_2) = \max(0, W_2 \cdot H_1 + b_2) \quad (6)$$

During the calculation performed over 1000 iterations, the ratio of test and train phases was set according to the Pareto principle (80% - 20%). The output vector containing the estimated values [21] is given in [Eq. 7] below.

$$Y_{pred} = W_3 \cdot H_2 + b_3 \quad (7)$$

The formulas used to determine the values for the weight coefficients and bias, which are iteratively determined during training [19], are given below.

$$W_{t+1} = W_t - \alpha \cdot \frac{\delta MSE}{\delta W_t} \quad (8)$$

$$b_{t+1} = b_t - \alpha \cdot \frac{\delta MSE}{\delta b_t} \quad (9)$$

As seen in [Eq. 8] and [Eq. 9], when setting new values for the weights and bias in each iteration, we find the effect of the previous values of the weights and bias on the multivariate continuous MSE function [7], [24] representing the mean square error by the chain rule.

For the optimization of the constructed neural network model, the adaptive moment estimation method (ADAM method) [17], which is an improved version of the stochastic sradient descent method (SGD method) that is resistant to gradient non-stationarity, was used. In this method, the predicted value of the first moment, i.e. the average gradient value – $\widehat{m}_t$, and the predicted value of the second moment, i.e. the mean of the squares of the gradients – MSG, are calculated in each iteration. With the following formula [Eq. 10], the optimal value of each parameter under consideration in the *i-th iteration* - f_i is found by taking into account the learning rate – η, the adaptive scaling factor based on the second moment of the gradient - $\sqrt{\widehat{v}_t} + \epsilon$, and the average gradient with reduced bias effects - $\widehat{m}_t$, and the MSG values in the considered formula [Eq. 10]. ($\epsilon -$ is a small constant to prevent division by zero, e.g.10^{-12}).

$$f_{i+1} = f_i - \frac{\eta}{\sqrt{MSG} + \epsilon} \widehat{m}_t \quad (10)$$

Thus, the neural network constructed in the research study under consideration has the following characteristics in Table 1.

Table 1. Characteristics of ANN

Structure of ANN	**Multi-Layer Perceptron (MLP)**
Raw dataset size	1230 x 8
Preprocessed dataset size	1230 x 5
Input layer neurons	5
Output layer neurons	5
Hidden layers	2
Neurons for each hidden layer	64
Activation function	ReLU
Epoch	1000
Train and test percentage	80%-20%
Optimisation method	ADAM method
Model performance evaluation	Mean Squared Error (MSE)

In this section, we have developed a comprehensive framework for estimating the biophysical parameters of the stochastic FitzHugh-Nagumo model using a multi-layer perceptron neural network. The proposed architecture consists of an input layer with five neurons corresponding to EEG frequency bands (delta, theta, alpha, beta, gamma), two hidden layers with 64 neurons each utilizing ReLU activation functions, and an output layer producing five parameter

estimates (a, b, c, σ_R, σ_P). The network was trained on a preprocessed dataset of 1230 EEG recordings from migraine patients with aura, employing the ADAM optimization method over 1000 epochs with an 80%-20% train-test split. The theoretical relationship between EEG features and model parameters, expressed through the ground truth formula, provides a physiologically motivated mapping that the neural network learns to approximate. The use of the Milstein method for numerical solution requires accurate parameter estimates, which our MLP regressor provides by minimizing mean squared error between predicted and true parameter values. The trained network achieved a mean squared error of 0.1475, demonstrating reasonable convergence and the ability to extract meaningful biophysical parameters from complex EEG signals. Having established the parameter estimation methodology, we now turn to the visual analysis of EEG data and the trained neural network's performance in the next section.

5. Visual representation and output of artificial neural network

The above-mentioned 1230x8 datasets were constructed based on the EEG tests of 1230 different patients [7], [18], and the `delta`, `theta`, `alpha`, `beta`, `gamma`, `age`, `gender` and `clinical_condition` feature vectors were constructed. In order to take into account biological variability and potential biases in the constructed dataset, as well as to normalize the data during the data preprocessing stage, the Z-score method was used. The condition of $z\_score > 3$ was set to exclude outlier data from the dataset, and we select an equal number of each age group to create a suitable balance in the `gender` and `age` columns. Finally, we fill in the empty values in the dataset with the $\sim ffill \sim$ method.

Based on the dataset prepared based on the EEG images of migraine patients with aura [20], visualized the EEG images of a migraine patient with an average aura in [Fig. 1]. According to the prepared dataset, abnormal dynamics are observed in the brain waves during the aura phases of migraine attacks in patients. Thus, while the brain waves change within specific normal ranges in the first 3 seconds of the EEG test [6], the upper and lower limits of these waves in the aura phases go out of the range of values [13].

There are many alternative approaches to parameter estimation in the considered stochastic differential equation system. Estimating parameters with simpler statistical models, for example, a linear regression model, is not considered convenient. Because [Eq. 1] is nonlinear, and also the relationship between brain waves based on EEG tests is nonlinear [8], [11] and is sensitive to biological bias. In particular, it is known that although the SVM model can be applied in the case of nonlinear dependencies, since the considered dataset is multidimensional and the considered model is stochastic, this method is not considered effective, since we cannot choose the kernel function [3] accurately and explicitly. However, the considered multilayer perceptron regression model uses optimization methods [12], [17] for bias and parameter estimation in each subsequent epoch. This helps us to find more accurate predicted values of the parameters.

Based on the neural network we built above, we can see the estimated values of the parameters in the system [Eq. 1] and the dynamics of the aura phase. Thus, we can substitute the estimated values of the parameters with the lowest mean square error rate in the system and solve the system of stochastic differential equations under consideration by the Milstein method.

In this section, we have presented the visual analysis of EEG data from migraine patients with aura and demonstrated the performance of our trained multi-layer perceptron neural network. The preprocessing pipeline, which employed Z-score normalization, outlier removal ($z_{score} >$

3), balanced sampling across age and gender groups, and forward-fill imputation for missing values, ensured data quality and reduced potential biases in the dataset. The visualization of average EEG test results clearly revealed the characteristic dynamics of migraine aura phases, showing that brain wave amplitudes during aura episodes deviate significantly from normal ranges observed in the initial phases of recording. While the first three seconds of EEG tests exhibited brain waves within physiologically normal bounds, the subsequent aura phases demonstrated abnormal upper and lower limit excursions across all frequency bands, confirming the pathological nature of cortical spreading depression. The trained MLP regressor successfully predicted parameter values with a mean squared error of 0.1475, demonstrating its capability to capture the nonlinear relationships between EEG frequency features and the underlying biophysical parameters of the stochastic FitzHugh-Nagumo model. The comparison between true and predicted outputs for test samples validated the effectiveness of our approach, though some discrepancies highlight the inherent complexity and variability in biological data. With the estimated parameters now available, we proceed to solve the stochastic differential equation system numerically to reproduce the membrane potential and recovery mechanism dynamics during migraine aura.

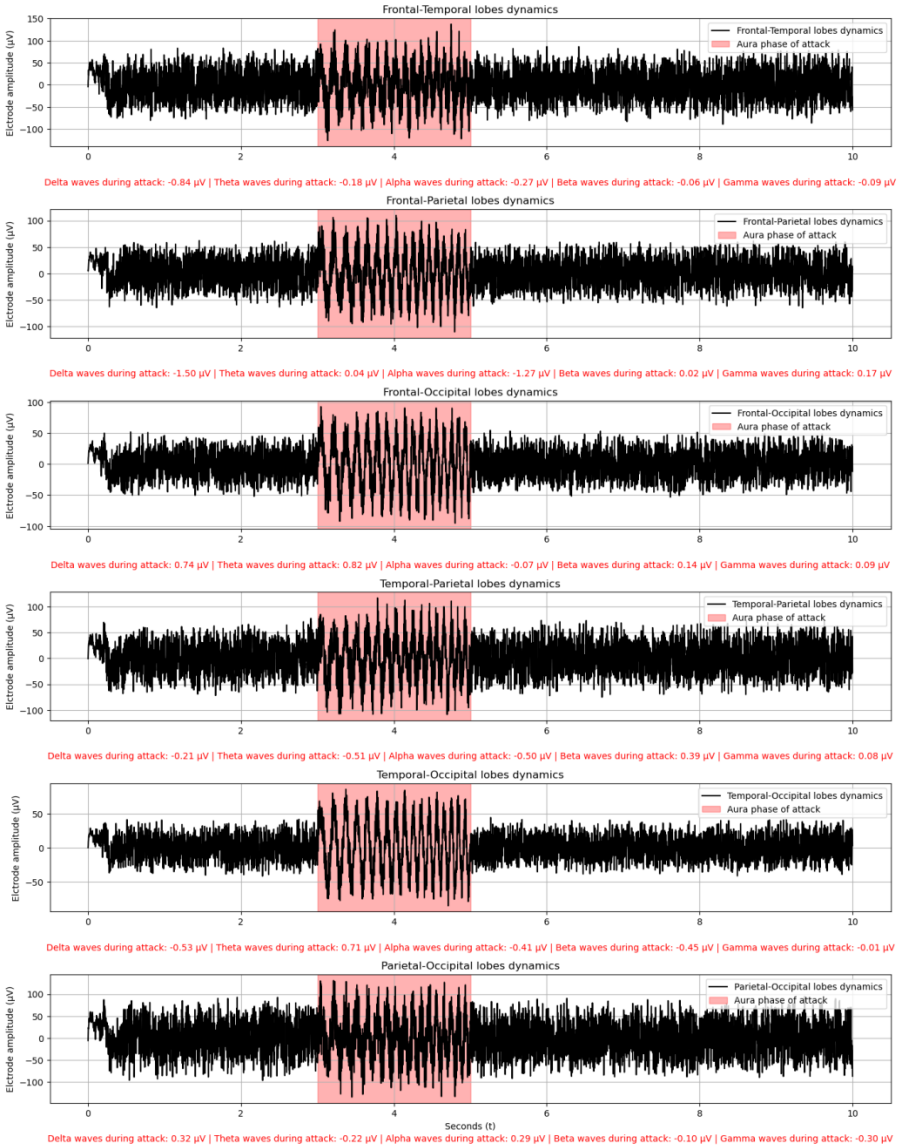

Fig 1. Average EEG Test result and aura phases during migraine attack.

Mean Squared Error value of MLP: 0.14754897042460963

Test label 1: True output: [0.37454012 0.95071431 0.73199394 0.59865848 0.15601864]

Predicted output: [0.60182965 0.09548657 0.44884072 0.55707213 0.39711602]

Test label 2: True output: [0.15599452 0.05808361 0.86617615 0.60111501 0.70807258]

Predicted output: [0.82112403 0.26972085 0.6645957 0.75420339 0.67455758]

6. Numerical solution of problem

The choice of numerical method for solving stochastic differential equations is crucial for obtaining accurate and reliable results. For the stochastic FitzHugh-Nagumo system under consideration, the Milstein method was selected over simpler approaches such as the Euler-Maruyama method due to several important advantages. The Euler-Maruyama method, while computationally simpler, only accounts for the first-order terms in the stochastic Taylor expansion and has a weak convergence order of 1.0 and strong convergence order of 0.5 (Kloeden & Platen, 1992). For the nonlinear stochastic system described in Equation (1), where both the drift and diffusion terms exhibit complex dependencies on state variables, the Milstein method achieves strong convergence of order 1.0, representing a substantial improvement over Euler-Maruyama (Gardiner, 2004).

Furthermore, the Milstein method preserves important statistical properties of the stochastic process more accurately than lower-order schemes, including the correct variance growth rate and correlation structure between membrane potential and recovery variables (Kloeden & Platen, 1992). This is essential for capturing stochastic resonance effects, where the interplay between deterministic dynamics and noise can enhance signal detection and information processing in neuronal systems (Wiesenfeld & Moss, 1995). Alternative higher-order methods, such as Runge-Kutta schemes for stochastic differential equations, would require significantly more function evaluations per time step without providing substantial accuracy improvements for our specific application.

The Milstein method improves the values of the functions sought at each subsequent step for both the deterministic and stochastic parts of the system during the numerical solution of stochastic differential equations [16]. It is known that we can write the obvious recurrence relation [Eq. 12] for the deterministic [Eq. 10] and stochastic [Eq. 11] parts of the system of stochastic differential equations under consideration.

$$\begin{cases} f_P(P,R) = P - \frac{P^3}{3} - R \\ f_P(P,R) = a + b \cdot P + c \cdot R \end{cases} \quad (10)$$

$$\begin{cases} g_P(P) = \sigma_P \\ g_R(R) = \sigma_R \end{cases} \quad (11)$$

$$\begin{cases} P_{n+1} = P_n + f_P(P,R) \cdot \tau + g_P(P_n)\Delta R_P + \frac{1}{2} g'_P(P_n) g_P(P_n)((\Delta R_P)^2 - \tau) \\ R_{n+1} = R_n + f_R(P_n, R_n) \cdot \tau + g_R(R_n)\Delta R_R + \frac{1}{2} g'_R(R_n) g_R(R_n)((\Delta R_R)^2 - \tau) \end{cases} \quad (12)$$

Here, as seen in [Eq. 13], the values of $(dR_P, dR_R))$ considered for each of the functions characterizing the membrane potential and recovery mechanism of neurons are selected according to the

values of a sufficiently small τ and a normally distributed random variable at each time instant.

$$\Delta R = \sqrt{\tau} \cdot \mathcal{N}(0,1) \qquad (13)$$

Thus, in [Fig. 2], an approximate solution of the stochastic FitzHugh – Nagumo differential equation system [Eq. 1] was found using the Milstein method, with initial conditions $P(0) = -1$ and $R(0) = 0$, step $\tau = 0.01$ and 1000 iterations, as a result of the numerical solution method, $P(T)$: $-$ 1.8213 and $R(T)$: 0.2182 were obtained, and the dynamics of the membrane potential and recovery mechanism in the aura phase were reflected.

7. Experimental results

For experimental results, we obtained comprehensive numerical results that validate our integrated modeling approach. The stochastic FitzHugh-Nagumo system was solved using the Milstein method with initial conditions $P(0) = -1$ and $R(0) = 0$, time step $\tau = 0.01$, and 1000 iterations corresponding to a total simulation time of $T = 10$ seconds.

Figure 2 illustrates the temporal dynamics of both the membrane potential $P(t)$ $(blue\ curve)$ and the recovery mechanism $R(t)$ $(orange\ curve)$ over the simulation period. The membrane potential exhibits characteristic oscillatory behavior with significant stochastic fluctuations, ranging approximately between -2.5 and -1.5, reflecting the excitable dynamics of neurons during migraine aura phases. The trajectory shows irregular oscillations superimposed on the deterministic limit cycle, which is consistent with the presence of stochastic resonance effects captured by the σ_P parameter. The recovery variable $R(t)$ demonstrates smoother dynamics with values fluctuating predominantly in the positive range between -0.5 and 1.5, indicating the slower time scale of recovery processes compared to membrane potential changes. The interaction between these two variables reproduces the characteristic depolarization-recovery cycles observed during cortical spreading depression. At the final time point $T = 10$ seconds, the membrane potential reached $P(T) = -1.8213$, indicating a state of partial depolarization relative to the initial condition, while the recovery mechanism attained $R(T) = 0.2182$, suggesting an active recovery state.

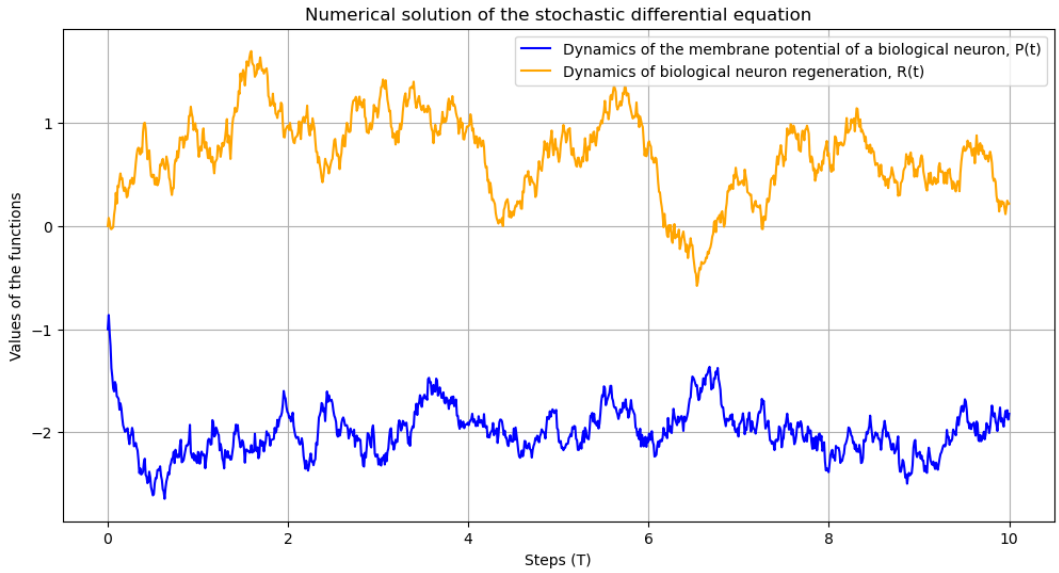

Fig 2. Solving a system of stochastic differential equations using the Milstein method

Final value of the membrane potential of a biological neuron P(T): -1.8213

Final value of the regeneration of a biological neuron R(T): 0.2182

8. Conclusion and future works

During the scientific research conducted based on the aura phases of EEG images during migraine attacks observed with aura, the stochastic FitzHugh – Nagumo differential equation system, which mathematically models migraine attacks, including electrode impulses of biological neurons, was examined and the predicted values of the positive internal self-regulation – a, the degree of influence of the membrane potential on the stochastic recovery mechanism – b and influence of neighboring neurons on the recovery mechanism – c parameters in the equation, including the stochastic resonance affecting the recovery mechanism and the stochastic resonance affecting the dynamics of the membrane potential parameters were found using a fully connected multi-layer perceptron regressor neural network.

The predicted values for the parameters were highly accurate with an error rate of $MSE \cong 0.14$ and reduced the potential error rate that could arise in each iteration when applying the Milstein method. When applying the Milstein method, the functions sought for the approximate solution of the nonlinear stochastic differential equation system under consideration were given initial approximation values of $P(0) = -1$ and $R(0) = 0$. At the end of 1000 iterations, the values of $P(T) = -1.8213$ and $R(T) = 0.2182$ were obtained.

Although the present study presents a novel approach for the diagnosis and parametric modeling of neurological diseases by integrating EEG data into the stochastic FitzHugh-Nagumo model, it has certain limitations. First, the study uses only averaged EEG signals and includes a more in-depth, time-frequency spectral analysis. Second, the selection of hyperparameters of the neural network model is based on experience and no automatic optimization algorithms are applied.

In terms of practical implications, this study supports the development of artificial intelligence-based methods for the early diagnosis of neurological diseases. In particular, it opens up new possibilities for modeling neural activity with EEG data.

Future directions include the use of medical based Big Data, the integration of time-frequency analysis, and the application of the model to real-time EEG signals. It is also planned to develop extended models for other neurological diseases. The results show that during migraine attacks with aura, the negative membrane potentials of biological neurons in the aura phase, and the function that evaluates the recovery mechanism receiving a sufficiently small value, confirm that the patient is experiencing conditions such as fatigue, physical weakness, motor disorders and short-term speech and hearing limitations.

Acknowledgement

This work was supported by the Institute of Data Science and Artificial Intelligence, also "AI-MED LAB" in the Institute of Biomedical Engineering under Azerbaijan Technical University

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