

UOT 539.3

DOI: <https://doi.org/10.30546/09085.2026.01.518>

INVESTIGATION OF STRESS STATE OF A COMPOUND CIRCULAR PLATE CONNECTED BY INTERFERENCE OF A DISK WEAKENED BY AN ECCENTRIC SQUARE HOLE

Vagif SHIRALIYEV^{1*}, Azer AHMADOV¹

vashiraliyev@beu.edu.az

¹Baku Engineering University, Khirdalan, Azerbaijan

ARTICLE INFO	ABSTRACT
Article history: Received:2025-08-21 Received in revised form:2025-09-03 Accepted:2025-10-01 Available online:2026-06-30 Keywords: complex variable function; conformably mapping function; pressure; Fourier series; conjugated functions. JEL Classification: L69, O31, O33	In this article is investigated stress state of a compound circular plates. A circular plate weakened by a central circular hole in which press – fitted a circular plate weakened by an eccentrically square hole. The joint subjected on an outer contour by partially – uniformly, and by inner contour uniformly pressure. The analytic function for multiply – connected finit domains presented as two pair of power functions with unknown coefficients which must satisfy definite boundary and contact conditions. For domain is different from the circular is used conformably mapping function. By substituting the regular functions in the boundary and contact conditions obtained eight infinite system of linear algebraic equations (ISLAE) for determining unknown coefficients of the regular functions. Once the regular functions are defined by formulas Kolosov – Muskhelishvili are determined stress state of the compound circular plates.

2521-6376/© 2026 The Author(s). Published by Baku Engineering University.

This is an open access article under the CC BY 4.0 license (<http://creativecommons.org/licenses/by/4.0/>).

1. INTRODUCTION

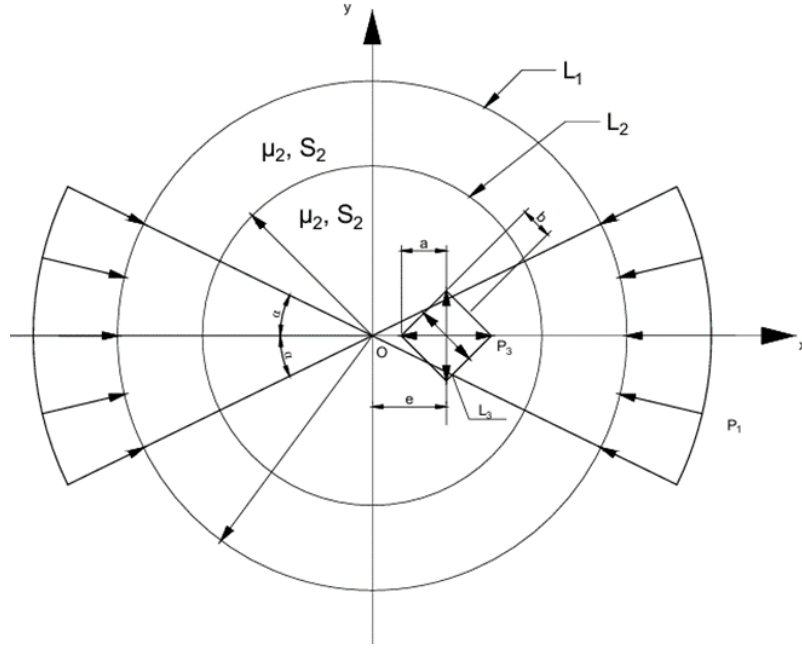
In the article is investigated stress state of compound plate. The compound plate consist of a circular plate of circular hole in which by interference a disk weakened by an eccentric square hole.

The material of plates characterizes with elastic constants μ_j and α_j ($j=1,2$) in which μ_j –the shearing modulus and ν_j is determined by the formula (Amenzade, 1996) (Bakhtiyarov, 1980).

$$\alpha_j = \frac{3-\nu_j}{1+\nu_j} \quad (1)$$

ν_j - the Poisson's coefficient.

A domain of the compound plate consist of two parts S1 and S2 (Bakhtiyarov & Ahmedov, 1985). The domain S1 bounded by concentric circles L1 and L2 and the domain S2 by concentric circle L2 and interior of a square hole (Kosmodamianskii, 1975). Besides the plate subjected on contour L1 to a partially uniformly pressure $p(t)$ and interior contour L3 by uniformly pressure $P3$ (Fig.1)



As is known, (Kuliyev, 2004), (Lavrentiev & Shabat, 1973) stress states of the compound plate are expressed by two functions $\varphi_j(z)$ and Ψ_j ($j=1,2$) of a complex variable $z = x + iy$. The functions maybe represented by the series

$$\varphi_j(z) = \sum_{k=0}^{\infty} A_k^{(j)} \left(\frac{z}{r_j}\right)^k + \sum_{k=1}^{\infty} B_k^{(j)} \left(\frac{A_j}{z - \varepsilon_j e}\right)^k \quad (2)$$

$$\Psi_j = \sum_{k=0}^{\infty} a_k^{(j)} \left(\frac{z}{r_j}\right)^k + \sum_{k=1}^{\infty} b_k \left(\frac{A_j}{z - \varepsilon_j e}\right)^k \quad (3)$$

where

$$A_j = |r_2| \text{ if } j = 1, A_j = |A| \text{ if } j = 2, E_j = \begin{cases} 0 & \text{if } j = 1 \\ 1 & \text{if } j = 2 \end{cases}$$

$A_k^{(j)}, B_k^{(j)}, a_k^{(j)}, b_k^{(j)}$ ($j = 1, 2$) - are unknown real coefficients which must be determined.

The partially – uniformly pressure subjected to exterior contour, taken as Fourier series, (Muskhelishvili, 1966), (Savin, 1968) i.e.

$$p(t) = \sum_{v=-\infty}^{\infty} \beta_v \left(\frac{t}{r_1}\right)^v, \quad (4)$$

Where,

$$\beta_v = -\frac{2P_1 r_1}{\pi(v-1)v} \sin(v-1)\alpha,$$

$$\beta_0 = -P_1 r_1 \cos\alpha,$$

$$\beta_1 = -\frac{2P_1 r_1}{\pi} \alpha,$$

$$v = \pm 1, \pm 3, \pm 5, \dots,$$

$$0 < \alpha \leq \frac{\pi}{2}$$

P_1 – intensity of external load

For domain S_2 we use Rimann's theorem (Timoshenko & Voynovskii-Krieger, 1966), (Timoshenko & Goodier, 1951) on conformal transformation:

Each simply connected domain S different from the total plane or from the plane with rejected (i.e. excluded) one point by means of an analytic function may be mapped one- to-one and conformal onto the interior or a unit disk. The precise solution of the problem on definition of a mapping function when domain is restricted by a square entails great difficulties. In order to simplify it we substitute a linear square for similar curvilinear square rounded vertices whose exterior is mapped onto exterior of a unit circle in the plane ξ by means of the function (Sherman, 1947).

$$z = A \left(\xi + \frac{m}{\xi^3} \right); A = \frac{a+b}{2}; m = \frac{1}{9} \quad (5)$$

Where a is a radius of a circle described around the square L_3 , b is a radius of circle in cribbed in the square. The linier square that we approximately substituted by a curvilinear square, nevertheless, differs from the latter, particularly, near the vertices corner points were substituted by rounded vertices.

The functions $\varphi_j(z)$ and $\Psi_j(z)$, ($j=1,2$) are regular on domains S_j ($j=1,2$) must be satisfy the following boundary and contact conditions:

$$\varphi_1(t) + t\overline{\varphi_1'(t)} + \overline{\psi_1(t)} = P(t) + C_1 \text{ on } L_1 \quad (6)$$

$$\varphi_2(t) + t\overline{\varphi_2'(t)} + \overline{\psi_2(t)} = -P_3(t - e) + C_3 \text{ on } L_3 \quad (7)$$

$$\varphi_1(t) + t\overline{\varphi_1'(t)} + \overline{\psi_1(t)} = \varphi_2(t) + t\overline{\varphi_2'(t)} + \overline{\psi_2(t)} \text{ on } L_2 \quad (8)$$

$$\frac{1}{\mu_1} (\alpha_1 \varphi_1(t) - t\overline{\varphi_1'(t)} - \overline{\psi_1(t)}) - \frac{1}{\mu_2} (\alpha_2 \mu_2 t - t\overline{\varphi_2'(t)} - \overline{\psi_2(t)}) = 2g(t) \text{ on } L_2 \quad (9)$$

in which $2g(t) = 2\delta r \left(\frac{t}{r} \right) \delta r$ – an interference of a displacement vector which is perpendicular to the line of connection; r – a radius of the disk.

t – affix point of any point of the considered L contour;

$\overline{\psi_j(t)}$ and $\overline{\varphi_j(t)}$ ($j = 1,2$) are adjoin functions for $\varphi_j(t)$ and $\psi_j(t)$ ($j = 1,2$) and $\overline{\varphi_1'(t)}$ and $\overline{\psi_1'(t)}$ their derivatives.

The boundary values of the functions $\varphi_1(z)$ and $\Psi_1(z)$ on L_1 and L_2 are expressed in the form:

$$\varphi_1(t) = \sum_{k=0}^{\infty} A_k^{(1)} \left(\frac{t}{r_1} \right)^k + \sum_{k=1}^{\infty} B_k^{(1)} \left(\frac{r_2}{r_1} \right)^k \cdot \left(\frac{r_1}{t} \right)^k \text{ on } L_1, \quad (10)$$

$$\psi_1(t) = \sum_{k=0}^{\infty} a_k^{(1)} \left(\frac{t}{r_1} \right)^k + \sum_{k=1}^{\infty} b_k^{(1)} \left(\frac{r_2}{r_1} \right)^k \cdot \left(\frac{r_1}{t} \right)^k \text{ on } L_1, \quad (11)$$

$$\varphi_1(t) = \sum_{k=0}^{\infty} A_k^{(1)} \left(\frac{r_2}{r_1} \right)^k \cdot \left(\frac{t}{r_2} \right)^k + \sum_{k=1}^{\infty} B_k^{(1)} \left(\frac{r_2}{t} \right)^k \text{ on } L_2, \quad (12)$$

$$\psi_1(t) = \sum_{k=0}^{\infty} a_k^{(1)} \left(\frac{r_2}{r_1} \right)^k \cdot \left(\frac{t}{r_2} \right)^k + \sum_{k=1}^{\infty} b_k^{(1)} \left(\frac{r_2}{t} \right)^k \text{ on } L_2, \quad (13)$$

Taking into account (10) – (13) we obtain expressions for functions $\overline{t\varphi'_1(t)}$ and $\overline{\psi_1(t)}$ on L_1 and L_2 respectively.

$$\overline{t\varphi'_1(t)} = 2A_2^{(1)} \sum_{k=1}^{\infty} \left[\left(\varepsilon_1(k)A_k^{(1)} - \varepsilon_2(k)(k-2)B_{k-2}^{(1)} \left(\frac{r_2}{r_1} \right)^{k-2} \right) \left(\frac{t}{r_1} \right)^k + (k+2)A_{k-2}^{(1)} \left(\frac{r_1}{t} \right)^k \right] \text{ on } L_1 \quad (14)$$

$$\psi_1(t) = a_0^{(1)} + \sum_{k=1}^{\infty} \left[a_k^{(1)} \left(\frac{r_1}{t} \right)^k + \left(\frac{t}{r_2} \right)^k + b_k^{(1)} \left(\frac{r_2}{r_1} \right)^k \left(\frac{t}{r_1} \right)^k \right] \text{ on } L_1 \quad (15)$$

$$\overline{t\varphi'_1(t)} = 2 \left(\frac{r_2}{r_1} \right)^2 A_2^{(1)} + \sum_{k=1}^{\infty} \left[\varepsilon_1(k) \left(\frac{r_2}{r_1} \right)^k A_k^{(1)} - \varepsilon_2(k)(k-2)B_{k-2}^{(1)} \left(\frac{t}{r_2} \right)^k + (k+2) \left(\frac{r_2}{r_1} \right)^{k+2} A_{k+2}^{(1)} \left(\frac{r_2}{t} \right)^k \right] \text{ on } L_2 \quad (16)$$

$$\psi_1(t) = a_0^{(1)} + \sum_{k=1}^{\infty} \left[a_k^{(1)} \left(\frac{r_2}{r_1} \right)^k \left(\frac{r_2}{t} \right)^k + b_k^{(1)} \left(\frac{t}{r_2} \right)^k \right] \text{ on } L_3 \quad (17)$$

The boundary value of the second member of the right portion of the $\varphi(z)$ and $\psi_1(z)$ on L_2 according to (2) and (3) is transformed by the power of variable t/r_2 , thus these functions on L_2 take a form

$$\varphi_2(t) = \sum_{k=0}^{\infty} A_k^{(2)} \left(\frac{t}{r_2} \right)^k + \sum_{k=1}^{\infty} H_k^{(2)} \left(\frac{r_2}{t} \right)^k \text{ on } L_2 \quad (18)$$

$$\psi_2(t) = \sum_{k=0}^{\infty} a_k^{(2)} \left(\frac{t}{r_2} \right)^k + \sum_{k=1}^{\infty} h_k^{(2)} \left(\frac{r_2}{t} \right)^k \text{ on } L_2 \quad (19)$$

Where,

$$H_v^{(2)} = \sum_{k=1}^v (-1)^{v-k} \cdot C_k^{v-k} \left(\frac{e}{r_2} \right)^{v-k} \left(\frac{A}{r_2} \right)^k \cdot B_k^{(2)};$$

$$h_v^{(2)} = \sum_{k=1}^v (-1)^{v-k} \cdot C_{-k}^{v-k} \left(\frac{e}{r_2} \right)^{v-k} \left(\frac{A}{r_2} \right)^k \cdot b_k.$$

Then for functions $\overline{t\varphi'_2(t)}$ and $\overline{\psi_2(t)}$ on L_2 we obtain

$$\overline{t\varphi'_2(t)} = 2A_2^{(2)} + \sum_{k=1}^{\infty} \left[\varepsilon_1(k)A_k^{(2)} - \varepsilon_2(k)A_k^{(2)} - \varepsilon_2(k)(k-2)H_{k-2}^{(2)} \left(\frac{t}{r_2} \right)^k + (k+2)A_{k-2}^{(2)} \left(\frac{r_2}{t} \right)^k \right] \text{ on } L_2 \quad (20)$$

$$\overline{\psi_2(t)} = a_0^{(2)} + \sum_{k=1}^{\infty} \left[a_k^{(2)} \left(\frac{r_2}{t} \right)^k + h_k^{(2)} \left(\frac{t}{r_2} \right)^k \right] \text{ on } L_2 \quad (21)$$

By analogy according to (2) and (3) we express functions $\varphi_2(z)$ and $\psi_2(z)$ on L_3 as an infinity series about powers of variable $t - e/A$. Then we will have

$$\varphi_2(t) = A_0^{(2)} + D_0^{(2)} + \sum_{k=1}^{\infty} \left[D_k^{(2)} \left(\frac{t-e}{A} \right)^k + B_k^{(2)} \left(\frac{A}{t-e} \right)^k \right] \quad (22)$$

$$\psi_2(t) = a_0^{(2)} + d_0^{(2)} + \sum_{k=1}^{\infty} \left[d_k^{(2)} \left(\frac{t-e}{A} \right)^k + b_k^{(2)} \left(\frac{A}{t-e} \right)^k \right] \quad (23)$$

Where,

$$D_v^2 = \sum_{K=E(v)}^{\infty} C_k^v \left(\frac{e}{r_2} \right)^k \left(\frac{A}{e} \right)^v, A_k^{(2)},$$

$$d_v = \sum_{K=E(v)}^{\infty} C_k^v \left(\frac{e}{r_2} \right)^k \left(\frac{A}{e} \right)^v, a_k^{(2)}$$

According to (22) we find,

$$\overline{\varphi_2'(t)} = \frac{D_1^{(2)}}{A} + \frac{1}{A} \left[(k+1) D_{k+1}^{(2)} \left(\frac{\bar{t}-e}{A} \right) - (k-1) B_{k-1}^{(2)} \left(\frac{A}{\bar{t}-e} \right)^k \right] \quad (24)$$

Boundary condition (7) on L_3 is transformed to a following form.

$$\varphi_2(t) + (t-e) \overline{\varphi_2'(t)} + e \overline{\varphi_2'(t)} + \overline{\psi_2(t)} = -P_3(t-e) + C_3 \quad (25)$$

By means of (5) and (22) we pass from variable $\left(\frac{t-e}{A} \right)$ to τ . Then

$$\sum_{k=1}^{\infty} D_k^{(2)} \left(\frac{t-e}{A} \right)^k = \sum_{k=1}^{\infty} D_k^{(2)} \sum_{v=-k}^{3k} * C_k^{\frac{v+k}{4}} m^{\frac{v+k}{4}} \frac{1}{\tau^v} \quad (26)$$

Taking into account that,

$$\sum_{v=-k}^{3k} * C_k^{\frac{v+k}{4}} m^{\frac{v+k}{4}} \frac{1}{\tau^v} = \sum_{v=k-4E\left(\frac{k}{4}\right)}^* C_k^{\frac{k-v}{4}} m^{\frac{k-v}{4}} \tau^v + \sum_{v=4E\left(\frac{k}{4}\right)+4-k} C_k^{\frac{v+k}{4}} m^{\frac{v+k}{4}} \frac{1}{\tau^v} \quad (27)$$

and

$$\sum_{k=1}^{\infty} \dots \sum_{v=k-4E\left(\frac{k}{4}\right)}^{k*} = \sum_{v=1}^{\infty} \dots \sum_{k=v}^{\infty} \dots,$$

$$\sum_{k=1}^{\infty} \dots \sum_{v=4-k+4E\left(\frac{k}{4}\right)}^{3k*} \dots = \sum_{v=1}^{\infty} \dots \sum_{k=\frac{v+4\alpha}{3}}^* \dots, \quad (28)$$

the formula (26) take a form

$$\sum_{k=1}^{\infty} D_k^{(2)} \left(\frac{t-e}{A} \right)^k = \sum_{v=1}^{\infty} \left[\alpha_v \cdot \tau^v + \beta_v \frac{1}{\tau^v} \right], \quad (29)$$

where,

$$\alpha_v = \sum_{k=v}^{\infty*} C_k^{\frac{k-v}{4}} m^{\frac{k-v}{4}} \cdot D_k^{(2)}; \beta_v = \sum_{k=\frac{v+4\alpha}{3}}^* C_k^{\frac{v+k}{4}} m^{\frac{v+k}{4}} \cdot D_k^{(2)} \text{ where } \alpha = 1, 2, 3 \dots$$

Symbol $E(k/4)$ signify an integrity part in $k/4$ and α – is as a smallest positive number, when $v + 4\alpha$ divided to 3 without remainder Asterisk over the series signify that in transforming to adjacent value k increased to 4.

By analogy a second expression (22) take a form

$$\sum_{k=1}^{\infty} B_k^{(2)} \left(\frac{A}{t-e} \right)^k = \sum_{v=1}^{\infty} \gamma_v \frac{1}{\tau^v} \quad (30)$$

where

$$\gamma_v = \sum_{k=1}^v * C_{-k}^{\frac{v-k}{4}} m^{\frac{v-k}{4}} B_k^{(2)}$$

Therefore, the value of the function

$$\varphi_2(t) = \varphi_2^*(\tau)$$

is reduced to a form

$$\varphi_2(t) = \varphi_2^*(\tau) = A_0^{(2)} + D_0^{(2)} + \sum_{v=1}^{\infty} \left[\alpha_v \tau^v + (\beta_v + \gamma_v) \frac{1}{\tau^v} \right] \quad (31)$$

By analogy we find expression for function

$$\psi_2(t) = \psi_2^*(\tau) = a_0^{(2)} + d_0^{(2)} + \sum_{v=1}^{\infty} \left[\alpha_v^* \tau^v + (\beta_v + \gamma_v) \frac{1}{\tau^v} \right] \quad (32)$$

By means of (31) and (30) we find expression for function $\overline{\varphi_2'(t)}$

$$\overline{\varphi_2'(t)} = \frac{D_1^{(2)}}{A} + \frac{1}{A} \sum_{v=1}^{\infty} \left[(L_v - M_v) \tau^v + E_v \frac{1}{\tau^v} \right] \text{ on } L_3 \quad (33)$$

By analogy (32) for function $\psi_2(t) = \psi_2^*(\tau)$ we obtain

$$\psi_2(t) = \psi_2^*(\tau) = a_0^{(2)} + d_0^{(2)} + \sum_{v=1}^{\infty} \left[\alpha_v^* \tau^v + (\beta_v^* + \gamma_v^*) \frac{1}{\tau^v} \right] \quad (34)$$

and

$$\overline{\psi_2(t)} = \psi_2^*(\tau) = a_0^{(2)} + d_0 + \sum_{v=1}^{\infty} \left[\alpha_v^* \frac{1}{\tau^v} + (\beta_v^* + \gamma_v^*) \tau^v \right] \quad (35)$$

where

$$\alpha_v^* = \sum_{k=v}^{\infty} * C_k^{\frac{k-v}{4}} m^{\frac{k-v}{4}} d_k^{(2)}; \beta_k^* = \sum_{k=\frac{v+4\alpha}{3}}^{\infty} * C_k^{\frac{v+k}{4}} m^{\frac{v+k}{4}} d_k^{(2)};$$

$$\gamma_v^* = \sum_{k=1}^v * C_{-k}^{\frac{v-k}{4}} m^{\frac{v-k}{4}} b_k^{(2)}; E_v = \sum_{k=v}^{\infty} * (k+1) C_k^{\frac{k-v}{4}} m^{\frac{k-v}{4}} D_{k+1}^{(2)};$$

$$L_v = \sum_{k=\frac{v+4k}{3}}^{\infty} * (k+1) C_k^{\frac{k+v}{4}} m^{\frac{k+v}{4}} D_{k+1}; M_v = \sum_{k=1}^v * (k-1) C_{-k}^{\frac{k-v}{4}} m^{\frac{k-v}{4}} D_{k-1}^{(2)}$$

Taking into account (5) and (34) for functions $(t-e)\overline{\varphi_2'(t)}$ and $e \cdot \overline{\varphi_2'(\tau)}$ we obtain (Shiraliyev, 1981)

$$\begin{aligned} (t-e)\overline{\varphi_2'(t)} &= (\tau \\ &+ \frac{m}{\tau^3}) \sum_{v=1}^{\infty} \left[\left(\varepsilon_1(v) D_v^{(2)} + L - M_v \right) \tau^v + E_v \frac{1}{\tau^v} \right] \\ &= \sum_{v=1}^{\infty} \left[\varepsilon_1(v) D_v^{(2)} + \varepsilon_4(k) (L_{v-1} - M_{v-1}) + (L_{v+3} - M_{v+3}) \right] \tau^v \\ &+ \sum_{v=1}^{\infty} \left[\varepsilon_1(v) (L_{v+1} - M_{v+1}) + \varepsilon_8(v) (L_{v-1} - M_{v-1}) + E_{v+1} + \varepsilon_3(v) m D_v^{(2)} \right. \\ &\left. + \varepsilon_5(v) E_{v-3} \right] \frac{1}{\tau^v} + E_1 + L_3 - M_3 \quad (36) \end{aligned}$$

and

$$e \overline{\varphi_2'(t)} = \frac{e}{A} D_1^{(2)} + \left(\frac{e}{A} \right) \sum_{v=1}^{\infty} \left[(L_v - M_v) \tau^v + E_v \frac{1}{\tau^v} \right] \quad (37)$$

2. COMPOSITION OF LINEAR ALGEBRAIC EQUATIONS.

FORMULA FOR STRESS STATE

Substituting (10), (14), (15) and (31), (36), (37), (35) with taking into account (5) in the boundary conditions (6) and (25) respectively and substituting (12), (16) – (18), (20), (21) in contact conditions (8) and (9) respectively and comparing coefficients in the same power variables, for determining unknown coefficients $A_k^{(i)}, B_k^{(i)}, a_k^{(i)}$ and $b_k^{(i)}$ ($i = 1, 2$) ($k = 1, 2, 3 \dots \infty$) we obtain the following eight infinity system of linear algebraic equations:

$$(1 + \varepsilon_1(k)) A_k^{(1)} - \varepsilon_2(k) (k-2) \left(\frac{r_2}{r_1} \right)^{k-2} B_{k-2}^{(1)} + \left(\frac{r_2}{r_1} \right)^k b_k^{(1)} = \beta_k \quad (38)$$

$$\left(\frac{r_2}{r_1} \right)^k B_k^{(1)} + (k+2) A_{k+2}^{(1)} + a_k^{(1)} = \beta_{-k}, \quad (39)$$

$$\alpha_k + L_k - M_k + \varepsilon_4(k) (L_{k-1} - M_{k-1}) + L_{k+3} - M_{k+3} + \beta_k^* + \gamma_k^* + \varepsilon_1(k) D_k^{(2)} = \varepsilon_1(k) A \cdot P_3 \quad (40)$$

$$\begin{aligned} \beta_k + \gamma_k + \alpha_k^* + E_k + E_{k+1} + \varepsilon_5(k) E_{k-3} + \varepsilon_3(k) \cdot m D_k^{(2)} + \varepsilon_1(L_{k-1} - M_{k+1}) \\ + \varepsilon_6(k) (L_{k-1} - M_{k-1}) = \varepsilon_3(k) m A P_3, \quad (41) \end{aligned}$$

$$\begin{aligned} (1 + \varepsilon_1(k)) \left(\frac{r_2}{r_1} \right)^{k+2} A_k^{(1)} + \varepsilon_2(k) (k-2) B_{k-2}^{(1)} + b_k^{(1)} - (1 + \varepsilon_1(k)) A_k^{(2)} + \varepsilon_2(k) (k-2) H_{k-2}^{(2)} - h_k^{(2)} \\ = 0 \quad (42) \end{aligned}$$

$$B_k^{(1)} + (k+2) \left(\frac{r_2}{r_1}\right)^{k+2} A_{k+2}^{(2)} + \left(\frac{r_2}{r_1}\right)^k a_k^{(1)} - H_k - \varepsilon_2(k)(k+2)A_{k+2}^{(2)} - a_k^{(2)} = 0 \quad (43)$$

$$(\chi_1 - \varepsilon_1(k)) \left(\frac{r_2}{r_1}\right)^k A_k^{(1)} + \varepsilon_2(k)(k-2)B_{k-2}^{(1)} - b_k^{(1)} - \gamma_{12}(1 - \varepsilon_1(k))A_k^{(2)} - \varepsilon_2(k)(k-2)\gamma_{12}H_{k-2}^{(2)} + \gamma_{12}h_k^{(2)} = \varepsilon_1(k)2\gamma_1\delta r, \quad (44)$$

$$\chi_1 B_k^{(1)} - (k+2) \left(\frac{r_2}{r_1}\right)^{k+2} A_{k+2}^{(1)} - \left(\frac{r_2}{r_1}\right)^k a_k^{(1)} - \gamma_{12}H_k^{(2)} + (k+2)\gamma_{12}A_{k+2}^{(2)} + \gamma_{12}a_k^{(2)} = 0 \quad (45)$$

For free members we will have

$$A_0 + 2A_2^{(1)} + a_0 - \beta_0 = C_1, \quad (46)$$

$$A_0^{(2)} + D_0^{(2)} + (e/A)D_1^{(2)} + a_0^{(2)} + d_0^{(2)} + E_1 + L_3 - M_3 = C_3, \quad (47)$$

$$A_0^{(1)} + 2\left(r_2/r_1\right)^2 A_2^{(1)} + a_0^{(1)} - A_0^{(2)} - 2A_2^{(2)} - a_0^{(2)} = 0, \quad (48)$$

$$\chi_1 A_0^{(1)} - 2\left(r_2/r_1\right)^2 A_2^{(1)} - a_0 - \chi_2 \gamma_{12} A_0 + 2\gamma_{12} A_2^{(2)} + \gamma_{12} a_0^{(2)}, \quad (49)$$

where,

$$\gamma_{12} = \gamma_1/\gamma_2$$

In order to determine $A_k^{(i)}, B_k^{(i)}, a_k^{(i)}$ and $b_k^{(i)}$ ($i = 1, 2$) on examples instead of solution infinity system of linear algebraic equations, is solved reduced system with $8N$ algebraic equations ($k=1, 2, \dots, N$) where N – is a number of restricted equations from every system, the choose which is dependent of the required correction of the solution.

Once determined approximate values of the required regular functions $\varphi_j(z)$ and $\psi_j(z)$ ($j = 1, 2$) by the following formula is determined stress components $\sigma_r^{(j)}, \sigma_\theta^{(j)}$ and $\sigma_{r\theta}^{(j)}$

$$\sigma_r^{(j)} + \sigma_\theta^{(j)} = 2[\varphi_j'(z) + \overline{\psi_j'(z)}], \quad (50)$$

$$\sigma_\theta^{(j)} - \sigma_r^{(j)} + 2i\sigma_{r\theta}^{(j)} = 2e^{2i\theta}[\bar{z}\varphi_j''(z) + \psi_j'(z)], \quad (51)$$

Where θ – an angle between external normal and axis Ox laid of from this axis.

3. Conclusion

For practical application obtained system of linear algebraic equations represented by the following compact form for easier use them in calculation

$$\sum_{i=1}^{8N} A_{ij}^{(4)} x_i = f_j^{(4)}, \quad (i, j = 1, 8N)$$

Where

$$x_n = A_n^{(1)}, x_{n+N} = B_n^{(1)}, x_{n+2N} = a_n^{(1)}, x_{n+7N} = b_n^{(1)}, x_{n+4N} = A_n^{(2)}, x_{n+5N} = B_n^{(2)}, x_{n+6N} = a_n^{(2)}, x_{n+7N} = b_{n+2}^{(2)}$$

Solution of the problem considered in the article represented by numerical examples (Table 1), on the basis which sketched stress diagrams for σ_r and σ_θ in case $P_1 = 0$ and $\frac{\sigma_r}{P_1}, \frac{\sigma_\theta}{P_1}$ in case $\delta_r = 0$ (Fig. 1 – 4). In case $P_1 = 0$, depending on the line of contact, determined radial pressure σ , and values compiled in Table 2 and their diagrams are shown for the case $\frac{\delta_r}{r} 10^3 = 1$ on the Fig.5

Table 1

Stress components N/m ²	Points of Z										Note
	R	0.5(R+r)	R-r	0.5(e+a+r)	e+a	e-a	-0.5r	-(R-r)	-0.5(R+r)	-R	
$-\sigma_r \cdot 10^{-6}$	0	8.54	42.73	38.79	0	0	37.09	46.71	9.12	0	$\frac{\delta_r}{r} \cdot 10^3 = 1$
$\sigma_\theta \cdot 10^{-6}$	16.27	25.01	58.93 -63	-69.23	-88.1	-90.46	-64.97	63.08 -53.9	26.24	17.24	
$-\frac{\sigma_r}{P}$	1.0	1.587	4.822	3.648	0	0	2.527	3.665	1.35	1.0	When $\delta_r = 0$ and $\alpha = 30^\circ$
$\frac{\sigma_\theta}{P}$	-0.108	-0.113	0.346 -3.892	-7.837	-23.06	-11.4	-4.986	0.947 -2.614	0.128	0.112	
Data	r/R = 0.4, e/r = 0.2, A/r = 0.36, e/A = 0.5556, m = 1/9, Materials:										
	Plate – copper ($\mu = 3.773 \cdot 10^{10} \frac{N}{m^2}, \chi = 2.008$), disk – steel ($\mu = 7.946 \cdot 10^{10} \frac{N}{m^2}, \chi = 2.2$),										

Table 2

$\frac{\delta_r}{r} \cdot 10^3$	$-\sigma_r \cdot 10^{-6} \frac{N}{m^2}$ – on the line of contact						
	$\Theta^0=0^0$	30^0	60^0	90^0	120^0	150^0	180^0
0.5	21.37	20.84	21.7	23.26	22.49	22.44	23.35
1.0	42.73	41.68	43.4	46.53	45.0	44.88	46.71
1.5	64.1	62.52	65.09	69.79	67.46	67.33	70.05
2.0	85.47	83.36	86.79	93.06	89.95	89.77	93.41
2.5	106.83	104.2	108.49	116.32	112.45	112.21	116.76
3.0	128.2	125.04	130.19	139.58	134.93	134.65	140.11

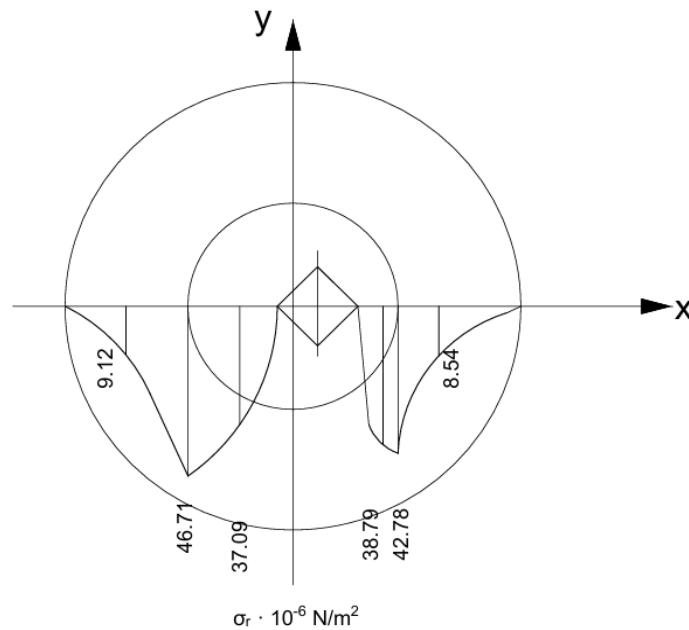


Fig. 2 Figure caption

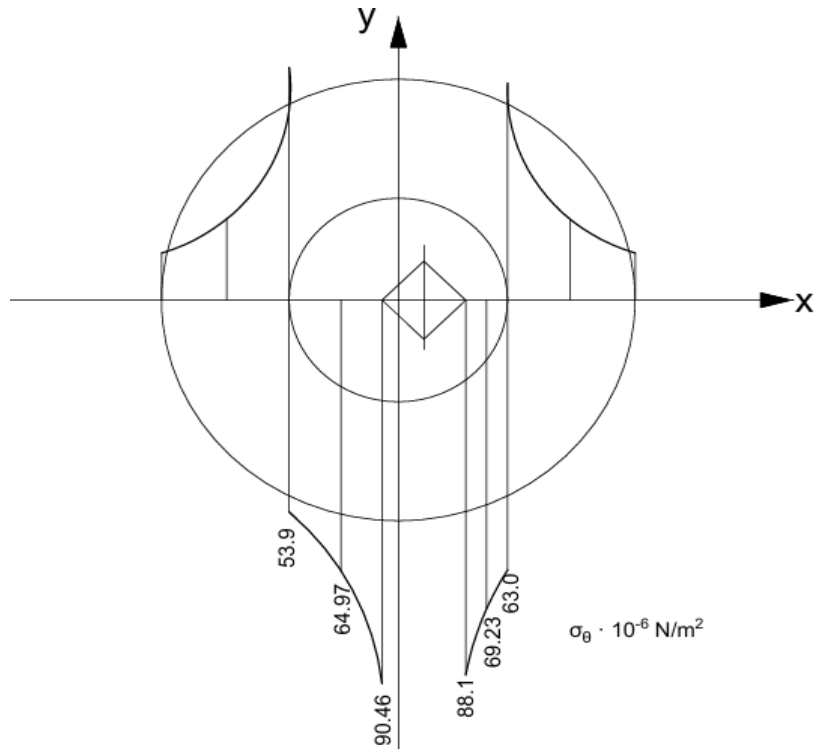


Fig. 3 Figure caption

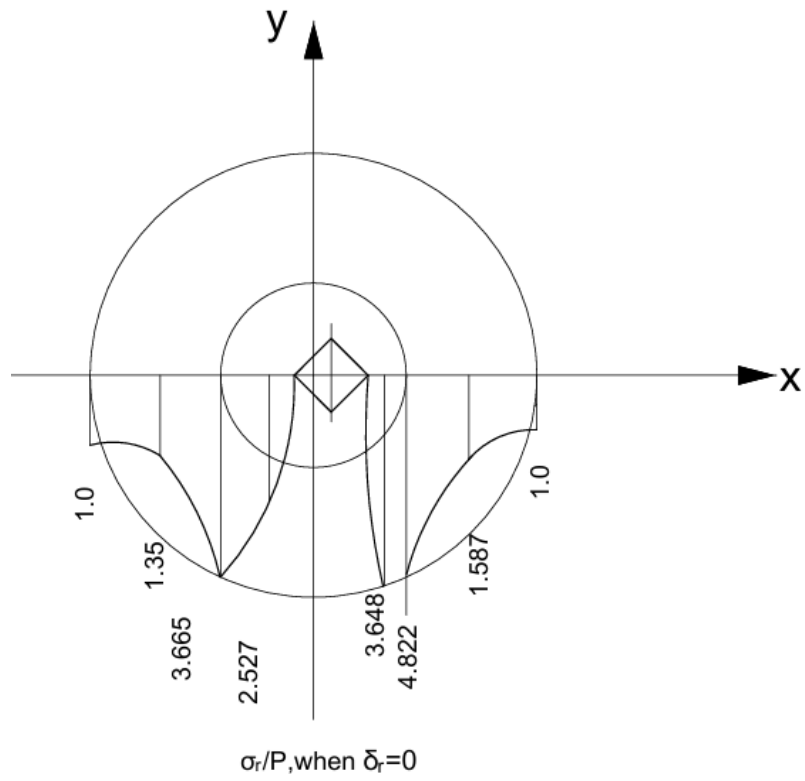


Fig. 4

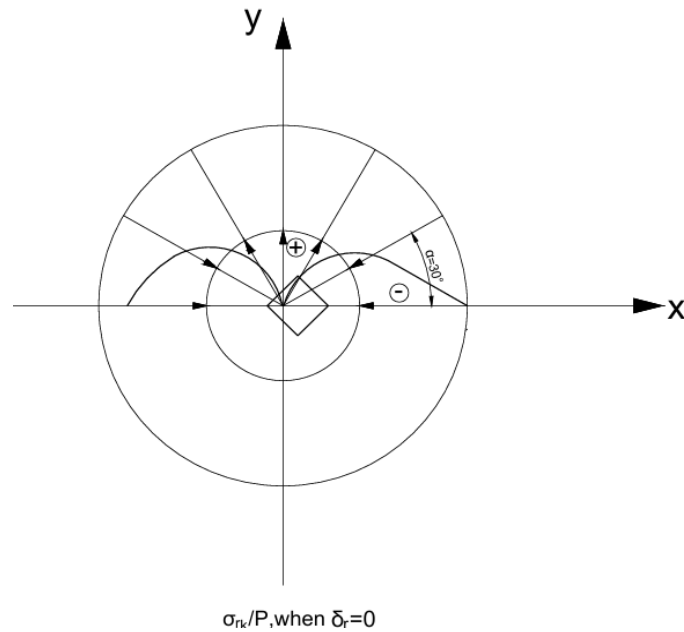


Fig. 5

REFERENCE LIST

- Amenzade, Y. A. (1996). Elasticity theory. Moscow: Vysshaya Shkola.
- Bakhtiyarov, I. A. (1980). Some problems of determining the stress state of a doubly connected plate. In Collection of scientific theses of institutions (pp. 3–9). Moscow.
- Bakhtiyarov, I. A., & Ahmedov, A. D. (1985). On the problem of determining the stress state of a circular compound plate connected by interference. In Proceedings of the VI Republic Conference of Young Researchers on Mathematics and Mechanics (pp. 78–82). Baku.
- Kosmodamianskii, A. S. (1975). Plane problems of elasticity theory for plates with holes, cuts, and bulges. Kiev: Vyshcha Shkola.
- Kuliyeu, S. A. (2004). Conformal mapping functions of complex domains. Baku: Azerneshr.
- Lavrentiev, M. A., & Shabat, B. V. (1973). Methods of the theory of functions of a complex variable. Moscow: Nauka.
- Muskhelishvili, N. I. (1966). Some basic problems of the mathematical theory of elasticity. Moscow: Nauka.
- Savin, G. N. (1968). Stress distribution near holes. Kiev: Naukova Dumka.
- Timoshenko, S. P., & Voynovskii-Krieger, S. (1966). Plates and shells. Moscow: Nauka.
- Timoshenko, S. P., & Goodier, J. N. (1951). Theory of elasticity. New York, NY: McGraw-Hill.
- Sherman, D. I. (1947). On a solution method for elasticity theory problems in doubly connected domains. Doklady Akademii Nauk SSSR, (8), 704–710.
- Shiraliyev, V. A. (1981). On some problems of elastic equilibrium of partially homogeneous plane bodies. In Proceedings of the III Republic Conference of Young Researchers on Mathematics and Mechanics (pp. 169–174). Baku: Elm..