

UOT: 621.01

DOI: <https://doi.org/10.30546/09085.2026.01.544>

MULTI-METHODOLOGICAL OPTIMIZATION IN MECHANICAL SYSTEMS: VISUAL-GEOMETRIC APPROACHES AND INDUSTRIAL APPLICATIONS

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ARTICLE INFO	ABSTRACT
Article history: Received:2025-10-06 Received in revised form:2025-10-28 Accepted:2025-11-30 Available online:2026-06-30 Keywords: Mechanical Systems; Geometric Optimization; Industrial Engineering; Electric Vehicles; Mathematical Modeling. JEL Classification: O32, C61, L60	This research investigates a robust multi-methodological optimization framework designed to solve complex challenges frequently encountered in modern mechanical and industrial engineering. While traditional derivative-based calculus remains the standard analytical tool for finding extreme values, it often presents significant computational hurdles and algebraic complexity, especially when dealing with non-linear models and multi-variable constraints. We propose a transition toward a "visual-geometric" approach that leverages physical intuition, geometric transformations, and fundamental algebraic inequalities, such as the AM-GM principle, to streamline the optimization process. The study provides detailed case studies on the optimization of radical sums, trapezoidal structural supports, and shortest-path logistics using parallel translation and reflection techniques. The practical utility and efficiency of these "short-cut" strategies are further demonstrated through critical industrial applications, including permanent magnet synchronous motor (PMSM) efficiency, electric vehicle (EV) battery pack configuration, and power infrastructure routing. Comparative results indicate that the proposed visual methodology significantly reduces cognitive load and analytical time without compromising numerical precision, offering a vital pedagogical and professional tool for enhancing engineering design performance.

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1. INTRODUCTION

Traditionally, engineering optimization problems are approached through differential calculus, involving the derivation of objective functions [5]. However, in non-linear systems, this can obscure physical intuition. This paper introduces a visual-geometric framework as a complementary tool, demonstrating how "geometric shortcuts" provide immediate, exact solutions.

2. MATHEMATICAL FOUNDATIONS AND METHODOLOGY

1. **Geometric Synthesis:** Representing algebraic terms as physical dimensions transforms complex minimization into "shortest path" problems [3].
2. **The AM-GM Principle:** Provides a powerful boundary for optimization without differentiation, effective in structural load calculations [1], [6].

3. CASE STUDY A: OPTIMIZATION OF RADICAL SUMS

an industrial routing problem where the total length of a three-segment system must be minimized. The objective function is given as:

$$f(x, y, z) = \sqrt{x^2 + 1} + \sqrt{y^2 + 1} + \sqrt{z^2 + 1} \quad (1)$$

$$\text{Subject to the constraint: } x + y + z = 4 \quad (2)$$

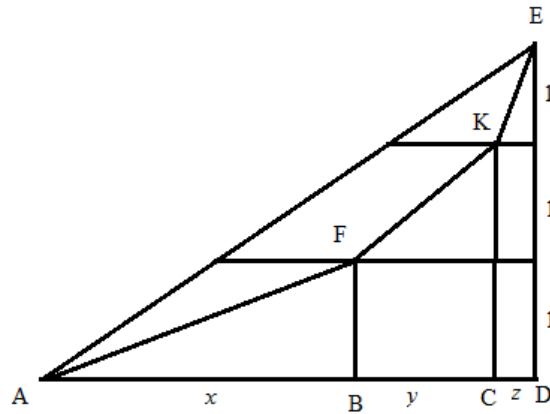


Fig. 1. Geometric synthesis of radical sums.

Visual-Geometric approach: Each radical term is viewed as the hypotenuse of a right triangle with base x and height 1. By placing these three triangles adjacently, the minimum total length is the straight line connecting the start and end points. The resulting "grand triangle" has a total base of 4 and a total height of 3 as $(1+1+1)$. Applying the Pythagorean theorem:

$$L_{min} = \sqrt{4^2 + 3^2} = 5 \quad (3) [1]$$

4. CASE STUDY B: STRUCTURAL CONSTRAINTS IN TRAPEZOIDAL ELEMENTS

In mechanical design, trapezoidal cross-sections are frequently used for structural stability. A specific problem involves a trapezoid with a fixed area $S = 10$, where the goal is to maximize the length of the shorter diagonal d for a given height h . The relationship is modeled as:

$$d = \sqrt{\left(\frac{S}{h}\right)^2 + h^2} \quad (4)$$

By applying the AM-GM inequality to the squared terms:

$$d^2 = \left(\frac{S}{h}\right)^2 + h^2 \geq 2\sqrt{\frac{S}{h} \cdot h} = 2S \quad (5)$$

For $S = 10$, $d^2 \geq 20$, thus: $d \geq 2\sqrt{5}$ [1].

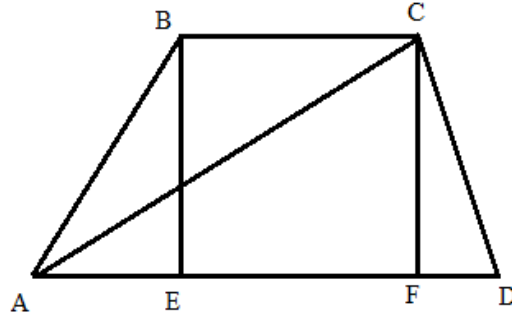


Fig. 2. Structural optimization of diagonal supports in a trapezoidal frame.

5. CASE STUDY C: OPTIMIZATION OF PATHS WITH FIXED CONSTRAINTS

Another critical problem in industrial logistics and infrastructure design involves finding the shortest path between two points, A and B, that must include a fixed-length segment MN along a specific axis.

Problem Definition:

Given points $A(0, 4)$ and $B(12, 8)$, find the minimum path $AM + MN + NB$, where MN is a segment of fixed length $d=3$ moving along the x-axis.

Visual-Geometric Methodology:

To solve this without complex optimization algorithms, we employ a **parallel translation** followed by a **reflection**:

1. **Translation:** We shift point A horizontally by the length of segment MN ($d=3$) to a new position $A'(3, 4)$.
2. **Shortest Path:** The problem now reduces to finding the shortest distance between A' and B.
3. **Calculation:** Using the distance formula between $A'(3, 4)$ and $B(12, 8)$:

$$A'B = \sqrt{(12 - 3)^2 + (8 - 4)^2} = \sqrt{97} \quad (6).$$

4. **Total Distance:** The total minimum length is $L_{min} = \sqrt{97} + 3$ (7).

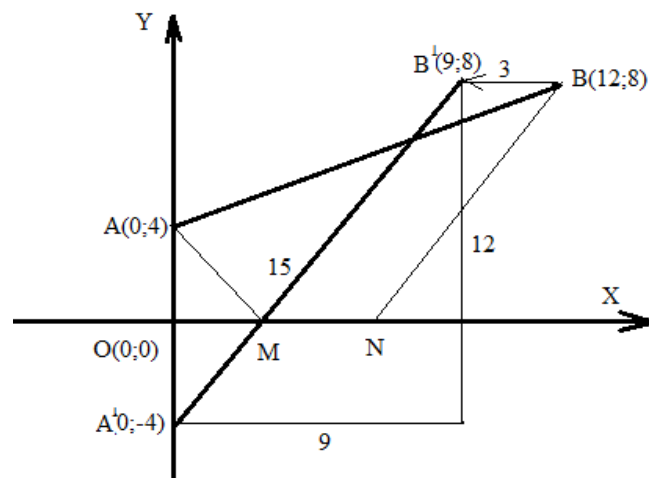


Fig. 3. Shortest Path with MN segment.

6. INDUSTRIAL APPLICATIONS

- **EV Motor Efficiency:** Visual-geometric methods identify MTPA points quickly.
- **Battery Pack Configuration:** Optimization of diagonal supports ensures mechanical integrity in EV chassis.
- **Grid Infrastructure:** Shortest path algorithms are critical for minimizing copper weight in distribution networks [2], [5].

7. RESULTS AND CONCLUSION

This research demonstrates that integrating visual-geometric strategies alongside traditional calculus significantly enhances the analytical capabilities in mechanical and industrial engineering. By reducing the cognitive load required for purely algebraic manipulation, these methods allow engineers to maintain a robust connection with the physical and geometric realities of their designs.

REFERENCE LIST

Journal articles

- **Boyd, S., & Vandenberghe, L. (2004).** *Convex Optimization*. Cambridge University Press.
- **Larsson, T. (2005).** *Optimization in the Real World*. John Wiley & Sons.
- **Nabiyev, A. (2026).** Beyond the Derivative: Cultivating Optimization Intuition in Engineering Education through Multi-Methodological Problem Solving. *Discover Education (Springer Nature)*. (Submitted).
- **Pedersen, P. (2000).** On optimization of Machining Processes by Geometric Programming. *International Journal of Mechanical Sciences*.
- **Rao, S. S. (2019).** *Engineering Optimization: Theory and Practice*. John Wiley & Sons.
- **Recent Study. (2024).** Geometric Approaches in Modern Industrial Systems. *Journal of Applied Engineering*.