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ON THE CALCULATION OF BRAKING TORQUES IN DRUM SHOE BRAKE MECHANISMS

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ABSTRACT

Friction brakes generate braking torque through the friction force between rotating (disk, drum) and non-rotating (pad, band) components. Drum-type brakes are primarily used in heavy-duty vehicles. To initiate braking, the shoe is pressed against the drum, creating contact pressure between the pressed surfaces. Tangential stresses arising from drum rotation produce the braking torque.

The braking torque in the brake mechanism depends on the contact pressure and the coefficient of friction between the compressed surfaces. With a large compressive force, the deformations of the drum and shoe differ, altering the drum's round cross-section and causing variations in braking force at different moments. Therefore, accurately determining contact pressure is crucial for drum brakes.

This article focuses on calculating braking torque by determining the deformations of the mold as close to real-world conditions as possible.

Introduction

In existing methods of calculating brake mechanisms, four laws of pressure forces are accepted a priori: constant $g = const$, sinusoidal $g = g_{max} sin\theta$, cosine $g = g_{max} cos\theta$ and square sinusoidal $g = g_{max} sin^2\theta$ by the length of the block. There is currently no single method for calculating brake parameters.

The pressure is distributed along the length of the shoe according to a sinusoidal law if the friction drum and brake shoes are absolutely rigid, the friction lining is ideally adjusted to the drum, and the deformation of the friction lining obeys Hooke's law. At high compression force, the deformations of the drum and shoe differ, changing the circular cross-section of the drum and causing changes in the braking force at different moments. The sinusoidal distribution law is typical for service braking; during braking with greater intensity, due to the increase in deformation of the shoes and the drum, which acquires an oval shape, it is distorted and approaches uniform [1].

The opposite of this is the cosine law, when the concentration of the specific load is observed in the middle part of the brake shoe, and an equal decrease in load is observed towards the edges of the advancing and escaping parts of the brake shoe. According to the statements presented in the work, the cosine law reflects the ovality of the rotating brake drum, which negatively affects the wear of the middle part of the shoe.

Determination of contact pressure

In fact, the true distribution of contact pressure is unknown in advance and must be determined from the solution of the contact problem for the lining - drum system. The material of the drum and the friction lining of the brake shoe is modeled by an elastic medium, the mechanical characteristics of which will be E_1, μ_1 and E_2, μ_2, respectively.

For any law of pressure distribution, the braking moment is determined through the resultant force of all elementary forces applied at a point whose coordinates are determined by the reduced radius and angle.

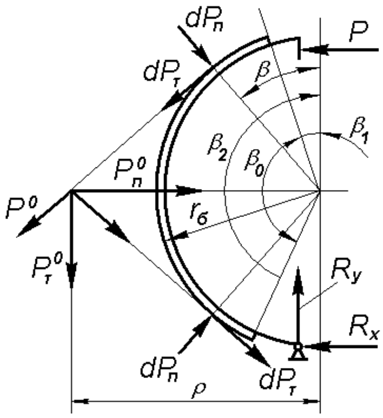

Figure 1. Diagram of forces acting on the shoe of a drum brake mechanism: where
p – is the pressure on the linings; dF – is the elementary area of the lining; b is the width of the lining;
r_b –is the radius of the drum; β – is the angular coordinate of the elementary area

The shoe is pressed against the brake drum under the action of force P. When the drum rotates in the direction indicated by the arrow, interaction forces arise between the drum and the shoe lining: the elementary normal force dP_n and elementary tangential force dP_τ.

To determine the total braking torque created by the shoe, it is necessary to know how the pressure changes along the length of the lining. As seen in the diagram, the resultant friction force (conditional) P_n^0 acts at a radius ρ, which depends on the angle β_0 = 90 – 120°. In calculations of braking torque, the resultant friction force is usually reduced to the radius of the brake drum, allowing the use of simplified formulas. However, this approach provides approximate values, which are insufficient for designing brake mechanisms. Therefore, we strive to determine the contact pressure more accurately by considering deformations and displacements during the drum-shoe contact in the braking process.

Let us assume that the friction lining is pressed into the inner surface of the drum over a certain section. A normal force (pressing force) is applied by an eccentric to each unit of length of the lining. It is assumed that the contact area extends across the entire width of the friction lining and does not change over time during braking. In the contact zone, in addition to normal forces (pressures) $g(\theta,\ t)$, tangential stress $\tau(\theta$, t) also acts

$$\begin{aligned} g(\theta,\ t) &= -\sigma_r\left(\theta,t,r_\delta\right); \\ \tau(\theta,\ t) &= \tau_{r\theta}\left(\theta,t,r_\delta\right), \end{aligned} \qquad (1)$$

associated with the contact pressure $g(\theta, t)$ according to Coulomb's law

$$g(\theta,\ t)\ =\ f\cdot g(\theta,\ t), \qquad (2)$$

where f is the coefficient of friction of the pair, "drum – lining". We assume that under the action of specific forces, $g(\theta,\ t)$ both $\tau(\theta,\ t)$ the drum and the friction lining are in a state of plane deformation.

Let us establish the relationships between the components of the displacements that occur in the contact area during vehicle braking [5-8]. We place the origin of the coordinates at the point of initial contact between the friction lining and the drum. Under the action of the pressing force, the lining will experience a displacement (settlement) of δ_2, and the drum will experience a displacement *of* δ_1. Point A, located on the surface of the lining, and point B, which comes into contact with it and is located on the inner surface of the drum, experience displacements of ν_1 and ν_2 in the radial direction, respectively, as a result of elastic deformation. Since the coordinates of points *A* and *B* become identical after they come into contact, this allows us to write the condition relating the displacements of the drum and the lining in the following form

$$\nu_1+\nu_2=\delta(\theta) \quad \left(\theta_0\le|\theta|\le\theta_1\right),$$

Here $\delta(\theta)$ *is* the settlement of the points of the surface of the drum and the lining, determined by the shape of the stamp base and the magnitude of the force F acting on it, θ_0, θ_1 – angles of the beginning and end of the pad coverage.

Let us now move on to finding the radial displacements v_1 and v_2.

Tangential forces $\tau(\theta,\ t)$ contribute to heat generation in the contact zone, with the total amount of heat per unit time being proportional to the friction power, and the amount of heat generated at a point in the contact area with coordinate θ is as follows

$$Q\left(\theta,\ t\right)=V\cdot\tau\left(\theta,\ t\right)=V\cdot f\cdot g\left(\theta,\ t\right), \qquad (3)$$

where $V=\omega r_b$ is the initial speed of movement of the drum surface points during braking; ω and r_b are the angular velocity and radius of the drum.

The amount of heat $Q(\theta, t)$ will be spent on the heat flow into the brake drum Q_* and a similar heat flow Q^* on increasing the temperature of the friction lining, i.e.

$$Q\left(\theta,\ t\right)=Q_*+Q^*. \qquad (4)$$

In [1-4, 9-10] it is recommended to use the average effective heat flow distribution coefficient in the calculation, which for one of the elements of the friction pair will be

$$\alpha_{TP2}=\left\{1+C_0\left\{\frac{F_{02}}{F_{01}}-\frac{F_{01}-F_{02}}{3F_{01}^2A}\left[\frac{1}{3}\ln\frac{\left(1/3-A\right)\left(1/3-F_{01}+A\right)}{\left(1/3+A\right)\left(1/3-F_{01}-A\right)}-A\ln\frac{3}{2}F_{01}\right]\right\}\right\}^{-1},$$

here

$$A=\sqrt{F_{01}^2+\frac{1}{9}};\qquad C_0=\frac{\Psi_{V2}b_2\lambda_1}{\Psi_{V1}b_1\lambda_2}.$$

For the other element of the pair

$\alpha_{TP1} = 1 - \alpha_{TP2}$.

Here $F_{01} = a_i t_T / b_i^2$ *is* the Fourier number; b_i is the thickness of the friction pair element; α_i is the thermal diffusivity coefficient; t_T – duration of a single braking in sec.; Ψ_{V1} – coefficient taking into account the effective volume participating in heat absorption; λ_i – thermal conductivity coefficient.

Based on the above, for the radial displacement v_1 – we will have [9, 10]

$v_1 = v_{1y} + v_{1T} + v_{1mp}$.

Here the term v_{1y} represents the elastic displacements of the points of the contact surface of the drum, v_{1T} - thermoelastic displacements caused by the temperature difference in the drum, term v_{1mp} represent movements caused by the removal of micro-protrusions from the inner surface of the drum.

To determine v_{1y} – it is necessary to solve the following auxiliary problem of elasticity theory on the inner surface of the drum at $\theta_0 \le |\theta| \le \theta_1$,

$$\begin{aligned} &\sigma_r = -g(\theta); \quad \tau_{r0} = 0 \quad at \quad r = r_b; \\ &\sigma_r = 0; \quad \tau_{r0} = 0 \quad at \quad r = R. \end{aligned} \tag{5}$$

Here *g(θ) is* an as yet unknown distribution function of contact stresses; *R* is outer radius of the drum.

To find v_{1r} it is necessary to solve the thermoelastic problem for a drum on the boundary of which there are no forces, and a heat source acts on the contact area.

3. Solution of the elastic problem

As is known [8], the stress state in an elastic body in the case of a plane problem in polar coordinates is determined by three stress components: σ_r, σ_θ, $\tau_{r\theta}$ which satisfy two equilibrium equations:

$$\begin{aligned} &\frac{\partial \sigma_r}{\partial r} + \frac{1}{r} \cdot \frac{\partial \tau_{r\theta}}{\partial \theta} + \frac{\sigma_r - \upsilon_\theta}{r} = 0; \\ &\frac{1}{r} \cdot \frac{\partial \sigma_\theta}{\partial \theta} + \frac{\partial \tau_{r\theta}}{\partial r} + \frac{2\tau_{r\theta}}{\tau} = 0; \\ &\sigma_r + \sigma_\theta = 4 \operatorname{Re} \Phi(z), \quad z = x + iy; \\ &\sigma_\theta - \sigma_r + 2i\tau_{r\theta} = 2\left[\bar{z}\Phi'(z) + \Psi(z)\right] e^{2i\theta}; \\ &\sigma_r - i\tau_{r\theta} = \Phi(z) + \bar{\Phi}(z) - e^{2i\theta}\left[\bar{z}\Phi'(z) + \Psi(z)\right]; \\ &2G(v_r + iv_0) = e^{-i\theta}\left[\chi\varphi(z) - z\bar{\Phi}(z) - \psi(z)\right], \\ &G = \frac{E}{2(1+\mu)}. \end{aligned} \tag{6}$$

Here Φ(z) and Ψ (z) are arbitrary analytic functions of the complex variable $z = x + iy$ in the region occupied by the elastic medium; E is the Young's modulus of the material; μ is the Poisson's ratio of the material; G is the shear modulus.

Using formulas (6), we write the boundary conditions of the elasticity theory problem in the following form

$$\Phi(z)+\bar{\Phi}(z)-e^{2i\theta}\left[\bar{z}\Phi'(z)+\Psi(z)\right]=0 \quad at \quad r=R;$$

$$\Phi(z)+\bar{\Phi}(z)-e^{2i\theta}\left[\bar{z}\Phi'(z)+\Psi(z)\right]=\begin{cases}-g(\theta) \ \ at \ \ r=r_\delta, \ \ \theta_0\le|\theta|\le\theta_1 \\ 0 \ \ at \ \ r=r_\delta, \ \ |\theta|\le\theta_0 \ \ and \ \ \theta_1\le|\theta|\le\pi\end{cases} \tag{7}$$

Expanding the unknown function $g(\theta)$ on the inner contour $r = r_\delta$ in a Fourier series, we will have

$$g(\theta)=A_0+\sum_{n=1}^{\infty}A_n\cos n\theta+\sum_{n=1}^{\infty}B_n\sin n\theta. \tag{8}$$

Here

$$A_n=\frac{1}{\pi}\int_{\theta_0}^{\theta_1}g(\theta)\cos n\theta d\theta; \ \ B_n=\frac{1}{\pi}\int_{\theta_0}^{\theta_1}g(\theta)\sin n\theta d\theta.$$

The expansion (2.3.5) can be rewritten as follows

$$g(\theta)=\sum_{k=-\infty}^{\infty}A'_k e^{ik\theta};$$

$$A'_0=\frac{A_0}{2}; \quad A'_k=\frac{A_k-iB_k}{2}; \quad A'_{-k}=\frac{A_k+iB_k}{2}.$$

We seek the solution to the boundary value problem in the form [70]

$$\Phi(z)=A\ln z+\sum_{k=-\infty}^{\infty}a_k z^k;$$

$$\Psi(z)=\sum_{k=-\infty}^{\infty}a'_k z^k.$$

Omitting the cumbersome calculations, we present the solution of problem (5) of elasticity theory

$$\begin{aligned}\sigma_r=&\frac{A_0}{\beta^2-1}\left(\frac{\beta^2}{\rho^2}-1\right)-\frac{\beta A_1}{\beta^4-1}\left(\frac{\rho}{\beta}-\frac{\beta^3}{\rho^3}\right)\cos\theta-\\&-\frac{\beta B_1}{\beta^4-1}\left(\frac{\rho}{\beta}-\frac{\beta^3}{\rho^3}\right)\sin\theta+\sum_{n=2}^{\infty}\frac{1}{2N}\left\{n\left[(n-1)-n\beta^2+\beta^{-2n}\right]\rho^{n-2}\right.\\&+n\left[(n+1)-n\beta^2-\beta^{2n}\right]\rho^{-(n+2)}+(n-2)\left[(n+1)-n\beta^{-2}-\beta^{-2n}\right]\rho^n+\\&\left.+(n+2)\left[(n-1)-n\beta^{-2}+\beta^{2n}\right]\rho^{-n}\right\}\times\left(A_n\cos n\theta+B_n\sin n\theta\right);\end{aligned} \tag{9}$$

$$\sigma_\theta = \frac{A_0}{\beta^2-1}\left(1-\frac{\beta^2}{\rho^2}\right) - \frac{\beta}{\beta^4-1}\left(\frac{3\rho}{\beta}+\frac{\beta^3}{\rho^3}\right)\left(A_1\cos\theta + B_1\sin\theta\right) + \\ +\sum_{n=2}^{\infty}\frac{1}{2N}\left\{n\left[-(n-1)+n\beta^2-\beta^{-2n}\right]\rho^{n-2} + \right. \\ n\left[-(n+1)+n\beta^2+\beta^{2n}\right]\rho^{-(n+2)} + (n+2)\left[-(n+1)+n\beta^{-2}+\beta^{-2n}\right]\rho^n + \\ \left. +(n-2)\left[-(n-1)+n\beta^{-2}-\beta^{-2n}\right]\rho^{-n}\right\}\times \\ \times\left(A_n\cos n\theta + B_n\sin n\theta\right); \tag{10}$$

$$\tau_{r\theta} = \frac{\beta}{\beta^4-1}\left(\frac{\rho}{\beta}-\frac{\beta^3}{\rho^3}\right)\left(A_1\sin\theta + B_1\cos\theta\right) + \\ +\sum_{n=2}^{\infty}\frac{1}{2N}\left\{n\left[-(n-1)+n\beta^2-\beta^{-2n}\right]\rho^{n-2} + \right. \\ n\left[(n+1)-n\beta^2-\beta^{2n}\right]\rho^{-(n+2)} + n\left[-(n+1)+n\beta^{-2}+\beta^{-2n}\right]\rho^n + \\ \left. +n\left[(n-1)-n\beta^{-2}+\beta^{2n}\right]\rho^{-n}\right\}\times\left(A_n\sin n\theta - B_n\cos n\theta\right); \tag{11}$$

$$v_r = -\frac{A_0}{\left(\beta^2-1\right)E}\left[(1-\mu)+(1+\mu)\frac{\beta^2}{\rho^2}\right]r_\delta\rho + \\ +\frac{1}{E}\left[\frac{m+1}{m}\cdot\frac{\beta^2}{\rho^2}\cdot c_2 + \frac{m-3}{m}\frac{\rho^2}{\beta^2}\cdot c_2\right]\beta r_\delta\cos\theta + \\ +\frac{1}{E}\left[\frac{m+1}{m}\cdot\frac{\beta^2}{\rho^2}\cdot c_2' + \frac{m-3}{m}\frac{\rho^2}{\beta^2}\cdot c_2'\right]\beta r_\delta\sin\theta + \\ +\sum_{n=2}^{\infty}\frac{1}{E}\left[-n\left(1+\frac{1}{m}\right)c_1''\left(\frac{\rho}{\beta}\right)^{n-1} + n\left(1+\frac{1}{m}\right)c_2''\left(\frac{\rho}{\beta}\right)^{-(n+1)} - \right. \\ \left. -\left(n-2+\frac{n+2}{m}\right)c_3''\left(\frac{\rho}{\beta}\right)^{n+1} + \left(n+2+\frac{n-2}{m}\right)c_4''\left(\frac{\rho}{\beta}\right)^{-(n+1)}\right]\beta r_\delta\times \\ \times\left(A_n\cos n\theta + B_n\sin n\theta\right); \tag{12}$$

$$v_\theta = \frac{1}{E}\left[\frac{m+1}{m}\cdot\frac{\beta^2}{\rho^2}\cdot c_2 + \frac{5m+1}{m}\frac{\rho^2}{\beta^2}\cdot c_2\right]\beta r_\delta\sin\theta - \\ -\frac{1}{E}\left[\frac{m+1}{m}\cdot\frac{\beta^2}{\rho^2}\cdot c_2' + \frac{5m+1}{m}\frac{\rho^2}{\beta^2}\cdot c_2'\right]\beta r_\delta\cos\theta + \\ +\sum_{n=2}^{\infty}\frac{1}{E}\left[n\left(1+\frac{1}{m}\right)c_1''\left(\frac{\rho}{\beta}\right)^{n-1} + n\left(1+\frac{1}{m}\right)c_2''\left(\frac{\rho}{\beta}\right)^{-(n+1)} + \right. \\ \left. +\left(n+4+\frac{n}{m}\right)c_3''\left(\frac{\rho}{\beta}\right)^{n+1} + \left(n-4+\frac{n}{m}\right)c_4''\left(\frac{\rho}{\beta}\right)^{-(n-1)}\right]\beta r_\delta\times \\ \times\left(A_n\sin n\theta - B_n\cos n\theta\right). \tag{13}$$

Here

$$\beta = \frac{R}{r_\delta}, \quad \rho = \frac{r}{r_\delta}, \quad m = \frac{1}{\mu};$$

$$c_2 = -\frac{A_1\beta}{2\left(\beta^4 - 1\right)}, \quad c_2' = -\frac{B_1\beta}{2\left(\beta^4 - 1\right)};$$

$$c_1'' = \frac{-\left(n-1\right) + n\beta^2 - \beta^{-2n}}{2\left(n-1\right)N}\beta^{-(n-2)};$$

$$c_2'' = \frac{-\left(n+1\right) + n\beta^2 + \beta^{2n}}{2\left(n+1\right)N}\beta^{-(n+2)}; \qquad (14)$$

$$c_3'' = \frac{-\left(n+1\right) + n\beta^{-2} + \beta^{-2n}}{2\left(n+1\right)N};$$

$$c_4'' = \frac{-\left(n-1\right) + n\beta^{-2} - \beta^{2n}}{2\left(n-1\right)N}\beta^{-n};$$

$$N = 2\left(n^2 - 1\right) - n^2\left(\beta^{-2} + \beta^2\right) + \left(\beta^{-2n} + \beta^{2n}\right).$$

CONCLUSION

To improve the accuracy of calculations for brake shoe-drum brake mechanisms, a method is proposed for determining the deformations of contacting surfaces and the contact stresses that arise in the shoe-drum pair during braking. Based on this approach, the problem of elasticity in the deformation of parts is addressed, and a method for the numerical calculation of contact stress is developed. Calculating contact stresses using modern computer technologies is straightforward.

In the future, to enhance the accuracy of calculations, it is proposed to consider thermoelastic deformations, displacements caused by the compression of micro-steps, and determine the total displacement of the contact points between the lining and the drum during braking. Determining the actual displacement will enable the development of a new approach to calculating drum-type brake mechanisms.

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