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SOLUTION OF THE THERMOELASTIC PROBLEM IN THE DRUM-SHOE CONTACT IN A BRAKE MECHANISM

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ARTICLE INFO	ABSTRACT
Article history: Received: 2025-09-12 Received in revised form: 2025-09-24 Accepted: 2025-10-25 Available online: 2026-06-30	During braking, the temperature of drum brakes can reach 300–350°C, which changes the coefficient of friction between the lining and the drum, causing the braking torque to fluctuate. The actual values of the braking torque, as well as the deformation of the shoe and drum, depend on the contact pressure, the temperature regime of the contacting surfaces, and their strength characteristics. Knowing the actual values of the contact pressure allows one to determine the deformation values; conversely, knowing the deformation values makes it possible to determine the contact pressure. On contacting surfaces, deformation can be represented by two components: elastic and thermoelastic deformation. Given the rigidity and strength properties, geometric dimensions of the brake mechanism elements, and the forces acting on the contact surfaces, it is possible to calculate the deformation values and contact stresses during braking. This article addresses the problem of determining thermoelastic deformations in order to calculate the braking torque of drum brake mechanisms.
Keywords: braking torque; drum; brake pads; thermoelastic deformations; contact pressure.	
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1. INTRODUCTION

Existing methods for calculating the braking torque of drum brakes, including the determination of braking torque, produce torque values that differ significantly from actual values, even when real friction coefficients are used. Current formulas for drum brake calculations are based on the conventional assumption of pressure distribution. At present, four pressure distribution laws at the drum–shoe interface are assumed a priori: uniform, sinusoidal, cosine, and square-sine. The results obtained from these approaches vary considerably. If the actual pressure on the contact surface is known, the braking force (tangential force) can be determined.

The real pressure distribution in a drum brake is typically determined experimentally. However, this process is time-consuming and costly, and accounting for the wide range of factors influencing these values is practically unfeasible.

The determination of elastic and thermoelastic deformations of the shoe and drum also requires further investigation. Existing methods for calculating the stiffness and strength of the brake drum are based on the assumption that the drum behaves as a tube or cylindrical shell subjected to uniform internal pressure. The shoe is commonly modeled either as a beam supported at its edges and loaded with a concentrated normal force or as a flat circular beam hinged at both ends.

In graphical analyses, all forces acting on the brake shoe are plotted according to a selected scale. In analytical approaches, the braking torque generated by the shoe is sometimes assumed to be infinitely large, causing the denominator in the formula to approach zero (Sakovich, N. Ye., et al., 2020). Solutions obtained under such assumptions fail to accurately reflect the real stress–strain state of the brake drum and shoes.

Any discrepancy between calculated and actual brake parameter values, regardless of direction, leads to unsatisfactory results. Therefore, the development of reliable and experimentally validated methods for drum brake calculation should be regarded as a critical issue, given its importance in the design of drum brake mechanisms.

A previously published study assumed that only elastic forces act on the contact surface (Namazov, B. F., 2024). This article extends that approach by also considering the influence of thermoelastic deformations caused by temperature increases on the friction surfaces.

2. MAIN PART

At the initial stage of calculating drum brake mechanisms, to determine the contact pressure on the drum-shoe interface, we considered the solution of an elastic problem without taking into account the temperature factor.

Since high-temperature heating and deformation occur on the friction surface of the drum and brake shoe during braking, it is crucial to study the influence of thermoelastic deformations on this contact surface in addition to elastic deformations (Chichinadze, A. V., et al., 1988). From this perspective, in the second stage of solving the problem, we study thermoelastic deformations.

To simplify the problem, we will assume that during braking we are dealing with a steady-state heat flow. Then, as is known, the temperature satisfies the equation of heat conduction theory

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \cdot \frac{\partial T}{\partial r} + \frac{1}{r^2} \cdot \frac{\partial^2 T}{\partial \theta^2} = 0 \quad (1)$$

under the following boundary conditions

$$\begin{aligned} \lambda \frac{\partial T}{\partial r} &= -q(\theta) \text{ at } \theta_0 \leq |\theta| \leq \theta_1, r = r_\delta \\ \lambda \frac{\partial T}{\partial r} + \alpha_1 (T - T_*) &= 0 \text{ at } |\theta| \leq \theta_0 \text{ and } \theta_1 \leq |\theta| \leq \pi, r = r_\delta \quad (2) \\ \lambda \frac{\partial T}{\partial r} + \alpha_2 (T - T_0) &= 0 \text{ at } r = R. \end{aligned}$$

Here α_1 – heat transfer coefficient from the inner cylindrical surface of the brake drum with temperature T_* ; α_2 – heat transfer coefficient from the outer cylindrical surface of the brake drum; $T(r, \theta)$ – temperature function. When solving the problem of heat conduction theory, it is assumed that heat exchange occurs on the outer surface of the brake drum with the external environment, which has a temperature T_0 . On the inner surface of the drum, on the contact area with the friction lining $\theta_0 \leq |\theta| \leq \theta_1$ there is a surface heat source with intensity $q(\theta)$, caused by external friction. On the rest of the drum surface $|\theta| \leq \theta_0$ and $\theta_1 \leq |\theta| \leq \pi$ heat exchange occurs with the environment having a temperature T_* (fig. 1).

The thermal conductivity coefficients of a material in the axial, circumferential, and radial directions are considered to be the same and independent of coordinates and temperature. The general solution to equation (1) is represented as

$$T(r, \theta) = 2 \operatorname{Re} F_1(z), \quad (3)$$

where

$$\begin{aligned} F_1(z) &= A \ln z + f_1(z); \\ f_1(z) &= \sum_{k=-\infty}^{\infty} b_k z^k, \end{aligned} \quad (4)$$

analytic function in a ring $r_\delta \leq |z| \leq R$; A – real constant.

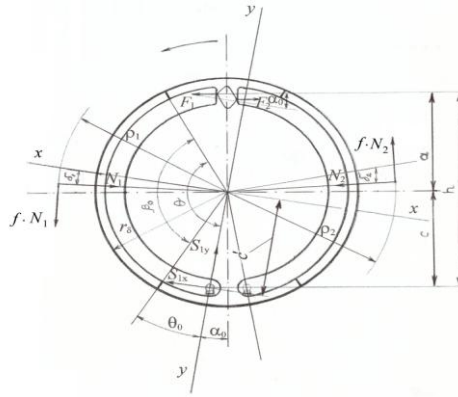


Fig. 1. Calculation diagram of the brake mechanism with fixed centers of rotation of the pads

For the function $f_1(z)$ we have boundary conditions

$$2 \operatorname{Re} [e^{i\theta} F_1'(z)] = \frac{\alpha_2}{\lambda} [T_0 - F_1(z) - \overline{F_1(z)}] \text{ at } r = R; \quad (5)$$

$$2 \operatorname{Re} [e^{i\theta} F_1'(z)] = \frac{\alpha_1}{\lambda} [T_* - F_1(z) - \overline{F_1(z)}] \text{ at } |\theta| \leq \theta_0 \text{ and } \theta_1 \leq |\theta| \leq \pi, \quad r = r_\delta; \quad (6)$$

$$2 \operatorname{Re} [e^{i\theta} F_1'(z)] = -\frac{1}{\lambda} q(\theta) \text{ at } \theta_0 \leq |\theta| \leq \theta_1, \quad r = r_\delta. \quad (7)$$

In the case where the heat transfer coefficients α_1 and α_2 large, unknown function $g(\theta)$ laid out on the inner contour $r = r_\delta$ in a Fourier series (8), (Namazov, B. F., 2024) can be replaced by the condition of constant temperature on the corresponding surfaces of the drum

$$f_1(t) + \overline{f_1(t)} + A \ln tt = T_0 \text{ at } t = \operatorname{Re}^{i\theta}; \quad (8)$$

$$f_1(t) + \overline{f_1(t)} + A \ln tt = T_* \text{ at } t = r_\delta e^{i\theta}, \quad |\theta| \leq \theta_0 \text{ and } \theta_1 \leq |\theta| \leq \pi; \quad (9)$$

$$\lambda \operatorname{Re} \left\{ e^{i\theta} \left[f_1'(z) + \frac{A}{z} \right] \right\} = -q(\theta) \text{ at } \theta_0 \leq |\theta| \leq \theta_1, \quad z = r_\delta e^{i\theta}. \quad (10)$$

Let on the contour $r = r_\delta$ ($t = r_\delta e^{i\theta}$) function $q(\theta) = \alpha_{III,1} V f g(\theta)$ expands into a complex Fourier series

$$g(\theta) = \sum_{k=-\infty}^{\infty} A'_k e^{ik\theta}. \quad (11)$$

In the designs of drum brakes of trucks, the actual conditions of heat exchange on the surface $r = r_\delta$, $|\theta| \leq \theta_0$ and $\theta_1 \leq |\theta| \leq \pi$ generally speaking, they are difficult, which is why the heat transfer coefficient is low. To simplify the problem, it can be assumed to be zero, i.e., it is assumed that on the inner surface of the drum $r = r_\delta$, $|\theta| \leq \theta_0$ and $\theta_1 \leq |\theta| \leq \pi$ there is no heat flow.

Based on the boundary conditions and relation (11) for determining $f_1(z)$ in the region occupied by the drum material, we have the following boundary conditions

$$f_1(t) + \overline{f_1(t)} + 2A \ln R = T_0, \quad t = R e^{i\theta} \quad (12)$$

$$\lambda \operatorname{Re} \left\{ e^{i\theta} \left[f_1'(\tau) + \frac{A}{\tau} \right] \right\} = -\alpha_{III,1} V f \sum_{-\infty}^{\infty} A'_k e^{ik\theta}, \quad \tau = r_\delta e^{i\theta}, \quad |\theta| \leq \theta_0 \quad \text{and} \quad \theta_1 \leq |\theta| \leq \pi. \quad (13)$$

$$\lambda \operatorname{Re} \left\{ e^{i\theta} \left[f_1'(\tau) + \frac{A}{\tau} \right] \right\} = 0, \quad \tau = r_\delta e^{i\theta}, \quad \theta_0 \leq |\theta| \leq \theta_1. \quad (14)$$

Let us remember that the coefficients A'_k are unknown to us at this time.

Substituting (5) into the boundary conditions (12 – 14) on the inner and outer surfaces of the drum and applying the procedure of the power series method, we find

$$2A \ln R + b_0 + \overline{b_0} = T_0; \quad (15)$$

$$b_k R^k + \overline{b_{-k}} R^{-k} = 0, \quad k \neq 0;$$

$$\frac{\lambda A}{2r_\delta} = -\alpha_{III,1} V f A'_0;$$

$$\frac{\lambda}{2} \left(b_1 r_\delta + \frac{A}{r_\delta} \right) = -\alpha_{III,1} V f A'_1; \quad (16)$$

$$\frac{\lambda}{2} k b_k r_\delta^{k-1} = -\alpha_{III,1} V f A'_k, \quad (k = \pm 2, \pm 3, \dots).$$

To obtain a solution to the thermoelasticity problem for a drum cross-section, we will use N.I. Muskhelishvili's dislocation analogy.

According to this analogy, the solution will consist of the solution to an "auxiliary problem," as if the body were not subjected to heating, but were subjected to a dislocation with the corresponding characteristics defined below, and a solution for the drum under the assumption of single-valued displacements. Let's consider the auxiliary problem. In our case, according to formula, we have

$$u^* + iv^* = 2 \int F_1(z) dz = 2 \left[Az \ln z - Az + b_{-1} \ln z + \sum_{k=-\infty}^{\infty} 'b_k \frac{z^{k+1}}{k+1} \right]. \quad (17)$$

The prime at the sum sign indicates that the index is excluded when summing $k = -1$.

In the case under consideration, the dislocation characteristics have the form

$$\begin{aligned}\varepsilon &= -\frac{2\pi v_1 A}{\lambda_* + \mu_1}; \quad \alpha_1^* + i\beta_1^* = 2b_{-1}; \\ \alpha_1^0 &= \frac{\pi v_1}{\lambda_* + \mu_1} \beta_1^*; \quad \beta_1^0 = \frac{\pi v_1}{\lambda_* + \mu_1} \alpha_1^*.\end{aligned}\quad (18)$$

Here λ_* and μ_1 – Lamé constants of the material. Complex “auxiliary problems” have the form (Muskhelishvili, N. I., 1977):

$$\begin{aligned}\Phi_*(z) &= B \ln z + a_0 + a_1 z + \frac{a_{-1}}{z}; \\ \Psi_*(z) &= \frac{a'_{-1}}{z} + \frac{a'_{-2}}{z^2} + \frac{a'_{-3}}{z^3}; \\ B &= \frac{\mu_1 \varepsilon}{\pi(1 + \chi_1)}; \\ a_0 &= \frac{\mu_1 \varepsilon}{2\pi(1 + \chi_1)} - \frac{\mu_1 \varepsilon (R^2 \ln R - r_\delta^2 \ln r_\delta)}{\pi(1 + \chi_1)(R^2 - r_\delta^2)}; \\ a_1 &= \frac{4\mu_1 \bar{b}_{-1}}{\pi(1 + \chi_1)(r_\delta^2 + R^2)}; \quad a_{-1} = \frac{2\mu_1 i b_{-1}}{\pi(1 + \chi_1)}; \\ a'_{-1} &= \frac{2i \bar{b}_{-1}}{\pi(1 + \chi_1)}; \quad a'_{-2} = \frac{2\mu_1 \varepsilon r_\delta^2 R^2}{\pi(1 + \chi_1)(R^2 - r_\delta^2)} \ln \frac{R}{r_\delta}; \\ a'_{-3} &= \frac{4\mu_1 i \bar{b}_{-1}}{\pi(1 + \chi_1)} \cdot \frac{R^2 r_\delta^2}{R^2 + r_\delta^2}.\end{aligned}\quad (19)$$

Here $\chi_1 = 3 - 4\nu_1$, ν_1 – Poisson's ratio of the material.

The solution to the problem for a drum with single-valued displacements was found earlier (Namazov, B. F., 2024).

For displacements in Cartesian coordinates in the case of a temperature difference, we have (Muskhelishvili, N. I., 1977):

$$u = u' + \frac{v_1 u^*}{2(\lambda_* + \mu_1)}; \quad v = v' + \frac{v_1 v^*}{2(\lambda_* + \mu_1)}. \quad (21)$$

Components u' , v' are found from the solution of the auxiliary problem (18)

$$\begin{aligned}u + iv' &= \frac{1}{2\mu_1} [\chi_1 \varphi_*(z) - z \overline{\Phi_*(z)} - \overline{\Psi_*(z)}]; \\ \varphi_*(z) &= \int \Phi_*(z) dz; \quad \psi_*(z) = \int \Psi_*(z) dz.\end{aligned}\quad (22)$$

Using relations (17), (18), (21), (22) after performing elementary, although somewhat cumbersome calculations, the radial displacement v_r we find it in the following form:

$$\begin{aligned}v_r &= v'_r + \frac{v_r v_r^*}{2(\lambda_* + \mu_1)}; \\ v'_r &= \frac{1}{2\mu_1} (c_0 + c_1 \cos \theta + c \sin \theta),\end{aligned}\quad (23)$$

$$\begin{aligned}
c_0 &= (\chi_1 - 1)Br \ln r + a_0 r + \frac{a'_2}{r}; \\
c_1 &= \chi_1 (a_0 - B) \ln r + \frac{\chi_1 \beta_1^* \mu_1 \ln r}{\pi(1 + \chi_1)} - \frac{\chi_1 \beta_1^* \mu_1 r^2}{\pi(1 + \chi_1)(r_\delta^2 + R^2)} - \frac{\mu_1 \beta_1^*}{\pi(1 + \chi_1)} + \\
&+ \frac{2\mu_1 r^2 \beta_1^*}{\pi(1 + \chi_1)(r_\delta^2 + R^2)} - \frac{\beta_1^* \ln r}{\pi(1 + \chi_1)} - \frac{\mu_1 \beta_1^* R^2 r_\delta^2}{\pi(1 + \chi_1)r^2(R^2 + r_\delta^2)}; \\
c_2 &= \frac{\chi_1 \mu_1 r^2 \alpha_1^*}{\pi(1 + \chi_1)(r_\delta^2 + R^2)} - \frac{\chi_1 \mu_1 \alpha_1^* \ln r}{\pi(1 + \chi_1)} - \frac{\mu_1 \alpha_1^*}{\pi(1 + \chi_1)} - \\
&- \frac{2\mu_1 r^2 \alpha_1^*}{\pi(1 + \chi_1)(r_\delta^2 + R^2)} + \frac{\alpha_1^* \ln r}{\pi(1 + \chi_1)} + \frac{\mu_1 R^2 r_\delta^2 \alpha_1^*}{\pi(1 + \chi_1)(R^2 + r_\delta^2)r^2}; \\
v_r^* &= Ar \ln r - Ar + \frac{1}{2} \alpha_1^* \ln r \cos \theta - \frac{1}{2} \beta_1^* \ln r \sin \theta + \\
&+ \sum_{k=1}^{\infty} \frac{r^k + 1}{k + 1} (\alpha_k \cos k\theta - \beta_k \sin k\theta); \\
b_k &= \alpha_k + i\beta_k.
\end{aligned} \tag{25}$$

3. CONCLUSIONS AND DISCUSSIONS

The conducted research makes it possible to formulate the main findings as follows. Existing methods for calculating drum brakes, which rely on established laws of contact pressure distribution, do not accurately reflect real operating conditions and produce varying results. Therefore, a new approach has been proposed and substantiated that more closely corresponds to the physical nature of the actual loads occurring during braking.

Solving the elastic contact problem between the shoe and drum in the brake mechanism constituted the initial stage of the analysis. In this study, thermoelastic problems were subsequently addressed. To obtain a solution to the thermoelasticity problem for a drum cross-section, N. I. Muskhelishvili's dislocation analogy was employed, and formulas for calculating radial displacements were derived.

Using the obtained radial displacement values, the elastic deformation results can be refined, making it possible to determine the actual braking torque for drum-shoe brake mechanisms in heavy-duty vehicles.

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