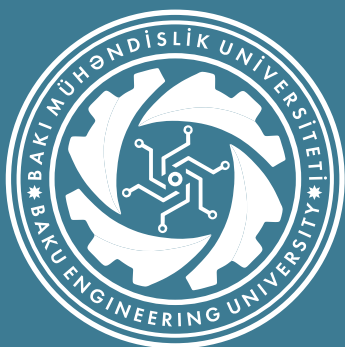




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STATIC ANALYSIS OF A FULLY OPEN GATE VALVE USING THE FINITE ELEMENT METHOD IN SOLIDWORKS

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ARTICLE INFO	ABSTRACT
Article history: Received:2025-10-23 Received in revised form:2025-11-12 Accepted:2025-12-11 Available online:2026-06- 30	<i>In this study, the static analysis of a fully open valve gate was carried out in SolidWorks Simulation using the Finite Element Method (FEM). The main objective of the research was to determine the distribution of stress, deformation, displacement, and safety factor of the gate under operating conditions. During the modeling process, the geometric shape and boundary conditions of the gate were defined according to its actual structural characteristics.</i>
Keywords: Finite Element Method (FEM); SolidWorks Simulation; Static Analysis; Valve; Stress.	
JEL Classification: C63; L60; O33	
	<i>A 3D model of the valve gate was created in SolidWorks, and a static analysis was performed on this model using the SolidWorks Simulation module. The mesh parameters were first defined (a fine mesh was selected), and the load corresponding to the real operating conditions of the gate was applied. As a result of the analysis, the distributed values of stress, deformation, and displacement along the surface of the gate were obtained.</i>
	<i>The conducted analysis allowed determining the distribution characteristics of stress and deformation, providing essential data for future structural optimization, material selection, and enhancement of operational durability.</i>

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1. INTRODUCTION

In modern industry, ensuring the safe and reliable operation of pipelines requires maintaining the high structural and mechanical integrity of one of their key components — the valves. Valves serve as essential control mechanisms in pipeline systems, responsible for regulating, preventing, or completely shutting off the flow of liquids and gases. The reliability and durability of these devices primarily depend on the strength properties and material resistance of the gate (slide plate).

In industrial applications, valves typically operate under high pressure, temperature, and mechanical loads, making it especially important to study the distribution of stress and deformation in their components — particularly in the gate section. Traditional analytical methods often fail to provide accurate results under such complex geometric and loading conditions, which is why the Finite Element Method (FEM) has become widely used in modern

engineering analyses. This method enables the modeling of the behavior of complex structures under load and provides results that closely approximate real operating conditions.

In this study, the static analysis of a fully open valve gate was performed using the SolidWorks Simulation environment. During modeling, the actual dimensions and material properties of the gate were taken into account, and the distribution of stress, deformation, displacement, and safety factor across various regions of the gate was analyzed.

Traditional analytical methods do not provide accurate analysis for structures with complex geometries and non-uniform loading conditions. Therefore, over the past decades, the Finite Element Method (FEM) has become a key tool for modeling and analyzing the behavior of such mechanical systems. This method allows the discretization of complex mechanical structures into smaller elements, enabling the simulation of real loading conditions in a computer environment based on the interaction between these elements.

The fundamental principles and theoretical basis of FEM have been thoroughly developed in study (1). That research demonstrates that the finite element approach allows for precise prediction of stress and deformation distributions in various materials and under different boundary conditions. The application of FEM in the design of mechanical structures is of great importance in terms of both safety and economic efficiency. In studies (2) and (3), the stress and deformation distributions in valve gate components used in industry were analyzed using FEM, and the results showed a high correlation with experimental observations.

Modern CAD/CAE software, including SolidWorks Simulation, has made the application of this method more practical and visually interpretable. The analysis modules available in the SolidWorks environment allow for the automatic calculation of stress, deformation, and safety factor, with the results displayed as graphical distribution maps, as described in detail in study (4). In study (5), the implementation of finite element analysis in mechanical structures and its advantages in computational accuracy were demonstrated. These studies show that modeling performed using modern engineering software enables a more comprehensive evaluation of real operating conditions compared to conventional analytical calculations.

Thus, the literature review indicates that static analyses conducted using the Finite Element Method (FEM) and programs such as SolidWorks Simulation are among the most effective approaches for assessing the reliability, safety level, and optimization potential of mechanical structures.

In this study, the static analysis of a fully open valve gate was carried out using the SolidWorks Simulation module. As a result of the modeling, the distributions of stress, deformation, displacement, and safety factor were examined, and the mechanical behavior of the gate was evaluated. The findings provide a valuable basis for future research aimed at improving the reliability, shape, and material optimization of such structural components.

2. STATIC ANALYSIS OF THE GATE

The simulation of the gate was carried out in SolidWorks according to the specified parameters. For this purpose, a 3D model of the gate was created, and mesh properties were applied. The pressure value on the surface of the gate was assumed to be equal to the valve operating pressure. As a result of the simulation, the distributed values of deformation, displacement, and stress were obtained (Figure 1).

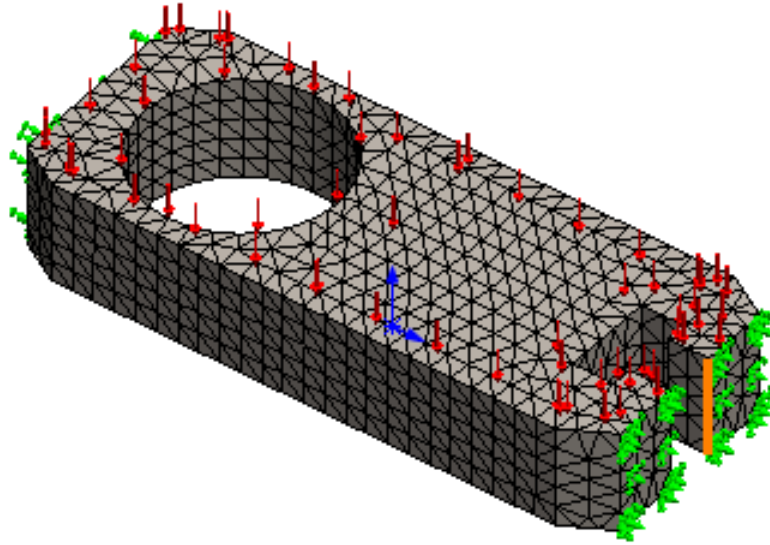


Fig. 1. Simulation of the Gate

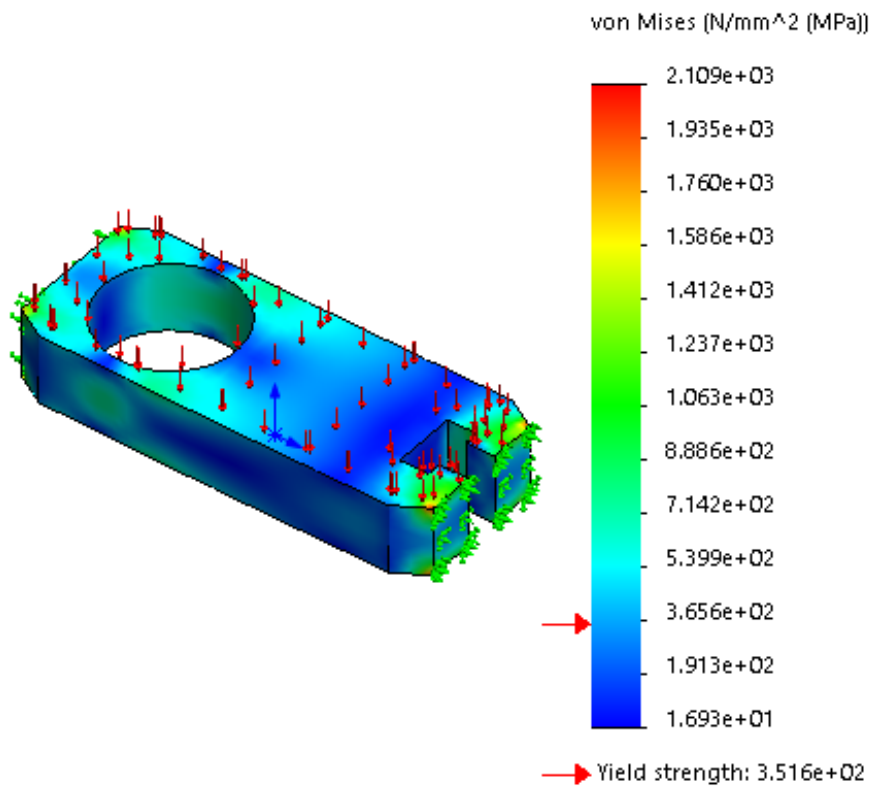
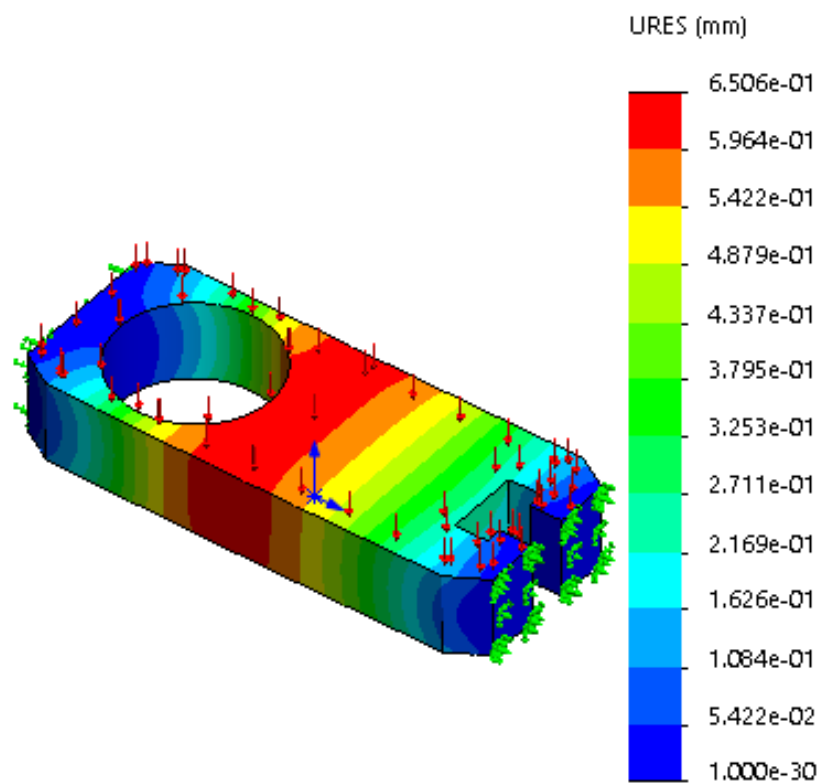
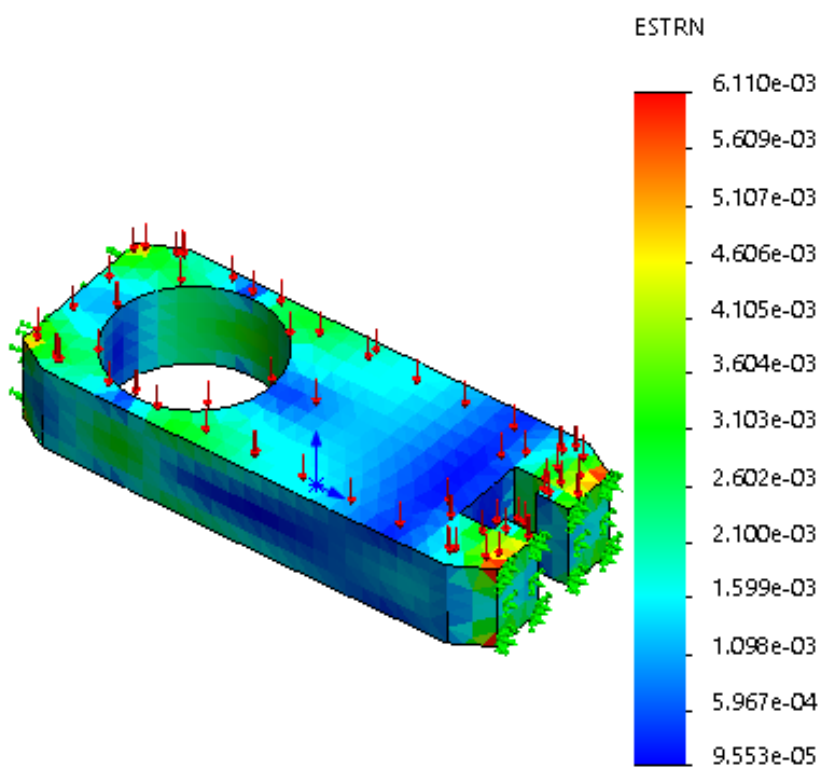


Fig. 2. Stress Distribution on the Gate Surface

The distribution of displacement on the gate surface is shown in Figure 3. As observed, the maximum displacement occurs at the central point of the gate (mid-cross-section). The highest deformation values are observed in the regions near the reinforced surfaces (Figure 3).



a)



b)

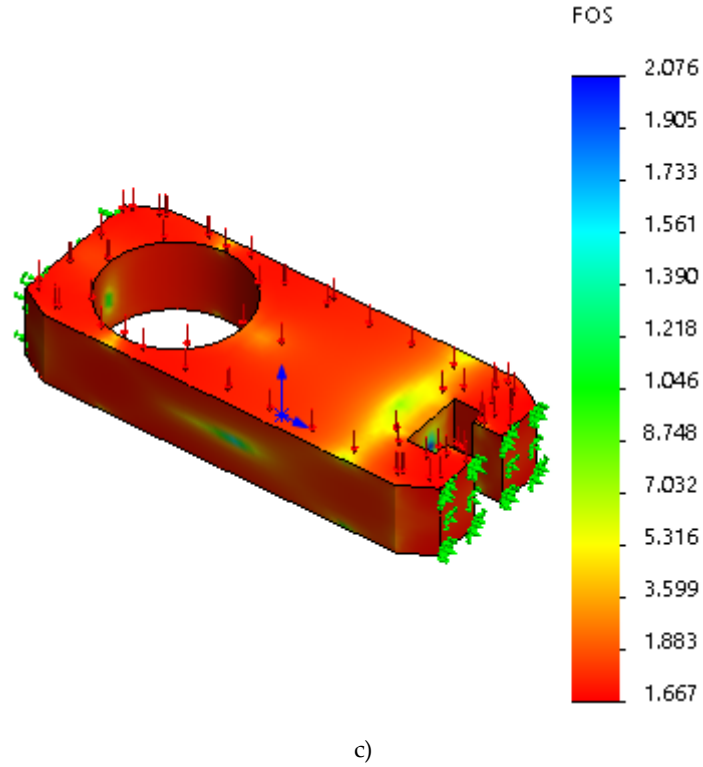


Fig. 3. Simulation Results: a) Displacement Results; b) Deformation Results; c) Safety Factor

The distribution of safety factor values is shown in Figure 5c. For brittle materials, the safety factor is calculated as the ratio of the ultimate stress to the applied working stress.

$$n = \frac{\sigma_h}{\sigma} \quad (1)$$

Here, n – is the safety factor, σ_h – is the ultimate stress, and σ – is the applied stress.

For ductile materials, the safety factor is calculated as the ratio of the yield strength to the applied stress. For the valve gate, the maximum safety factor was found to be 2, while the minimum value was 1.66. The highest safety factor values were observed near the spindle connection point, at the central surface, and at the bottom surface of the seat.

3. CONCLUSION AND DISCUSSIONS

1. Modeling based on the Finite Element Method (FEM) showed that the stress distribution in the gate is non-uniform, with the main load concentrated at the spindle contact area. This zone is potentially exposed to maximum stress and fatigue risk.
2. These distributions varied according to the stiffness and geometric characteristics of the gate, with maximum values observed on the top and bottom surfaces and minimum values at the central surface. This allows the identification of the primary elastic behavior direction and potential weak points of the gate.
3. The calculated safety factors remained within the material's elastic limit, indicating that the gate can operate reliably under static loading conditions. The analysis confirms that the material selection and structural design are appropriate.

4. Based on the results, certain modifications can be made to the gate's structure to reduce local stresses and optimize material usage. This can enhance mechanical strength and reduce manufacturing costs. These findings provide a practical basis for future studies aimed at optimizing valve components, increasing load capacity, and minimizing energy losses.
5. Overall, the analysis demonstrated that FEM-based simulation in SolidWorks is a reliable, efficient, and cost-effective tool for modern engineering evaluations.
6. It is advisable to conduct thermomechanical and dynamic analyses of the gate, as well as to test different material types and optimized design variants to further improve performance.

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IMPROVING THE MANUFACTURING PROCESS AND INCREASING THE WEAR RESISTANCE OF SHUT-OFF VALVES IN THE OIL AND GAS INDUSTRY

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ARTICLE INFO	ABSTRACT
Article history: Received:2025-11-02 Received in revised form:2025-11-14 Accepted:2025-12-15 Available online:2026-06-30 Keywords: gate valve; pipeline valves; corrosion resistance; titanium alloys; durability JEL Classification: L60; L71; O33	This study presents the design of a combined-type slide gate valve incorporating titanium alloy locking elements to enhance the corrosion and wear resistance of shut-off valves used in the oil and gas industry. The purpose of this work is to improve the operational reliability and durability of gate valves under harsh production and transportation conditions typical for Azerbaijan's oil fields. Modern shut-off valves must ensure not only reliability and continuous operation but also satisfy key design criteria such as full-bore configuration to minimize hydrodynamic resistance, laminar flow conditions, and the absence of stagnant zones that may impede valve closure or accelerate failure. The paper analyzes existing valve types and identifies the gate valve as a critical component in pipeline safety systems. Furthermore, it proposes an innovative structure using titanium-based components to mitigate wear and corrosion at critical contact interfaces. The proposed design aims to extend the service life of valves, reduce maintenance costs, and enhance safety performance in hydrocarbon transport infrastructure.

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1. INTRODUCTION

The operational reliability and cost efficiency of hydrocarbon extraction, transportation, and processing in the oil and gas industry are determined by the technological advancement and reliability of shut-off valves. These valves operate under extreme conditions, requiring high performance across a broad range of pressures, temperatures, and environments, including exposure to aggressive fluids and gases, seismic loads, and humidity fluctuations. Environmental protection and personnel safety are key priorities for the industry, thereby increasing the design and performance demands for shut-off valves.

Today, several main types of shut-off valves are commonly used in oil and gas systems: globe valves, gate valves, butterfly valves, and ball valves. Each type offers specific advantages and limitations. For example, **globe valves** are inexpensive and easy to manufacture but suffer from high hydraulic resistance and one-directional flow. **Butterfly valves** operate effectively only within a narrow temperature and pressure range. **Ball valves** provide excellent tightness, quick

operation, and low maintenance costs, but their manufacturing is costly, and they cannot be repaired outside factory conditions.

In contrast, **knife gate valves** possess numerous benefits: simplicity, compact design, low cost, and wide applicability across varying operational environments. Among these, the gate valve remains the most widely used device for fluid shut-off in oil and gas pipelines.

2. MAIN DISCUSSION

In oil and gas systems, the reliability of hydrocarbon production and transportation processes largely depends on the performance of technologically advanced and durable shut-off valves. These devices must operate safely under extreme pressure, temperature, and environmental conditions (including humidity, seismic activity, and aggressive fluids).

The main categories of shut-off valves include **globe valves**, **gate valves**, **butterfly valves**, and **ball valves**, each with specific characteristics:

- **Globe valves** are low-cost and easy to manufacture but feature high hydraulic resistance and can handle flow in only one direction.
- **Butterfly valves** are suitable for narrow temperature and pressure ranges.
- **Ball valves** offer excellent tightness and low maintenance but cannot be repaired in the field and fail above 2000°C.
- **Knife gate valves**, however, offer several advantages, including simplicity, small size, cost efficiency, and adaptability to various working conditions.

This paper focuses on **gate valves**, which are widely used to shut off the working fluid in oil and gas systems. The movable part—the gate—is positioned perpendicular to the fluid flow, and its movement controls the passage opening or closure. The actuator force is transmitted through the spindle mechanism.

A typical gate valve comprises the following key components: **body**, **gate**, **stem (post)**, **sealing elements**, and **gland packing** (Fig. 1). Tight sealing in the closed position is achieved by pressing the gate against the seat under pressure from the working medium. Gate valves are designed with either single- or double-sided sealing and may include rising or non-rising stems.

The valve body, usually rectangular or cylindrical, can be cast as a single piece or welded from multiple sections. It may include features such as a removable bottom cover and through-gate design. Body materials range from **gray or ductile cast iron** to **carbon, low-alloy, and stainless steels**, as well as **non-ferrous alloys (e.g., aluminum)** and **non-metallic materials (e.g., PVC)**.

The **gate** may be a plate made from rolled steel with parallel sealing surfaces or a sharpened “knife edge” for cutting through viscous or contaminated fluids. Polished stainless steel gates ensure smooth sliding against sealing materials, minimizing friction and preventing jamming.

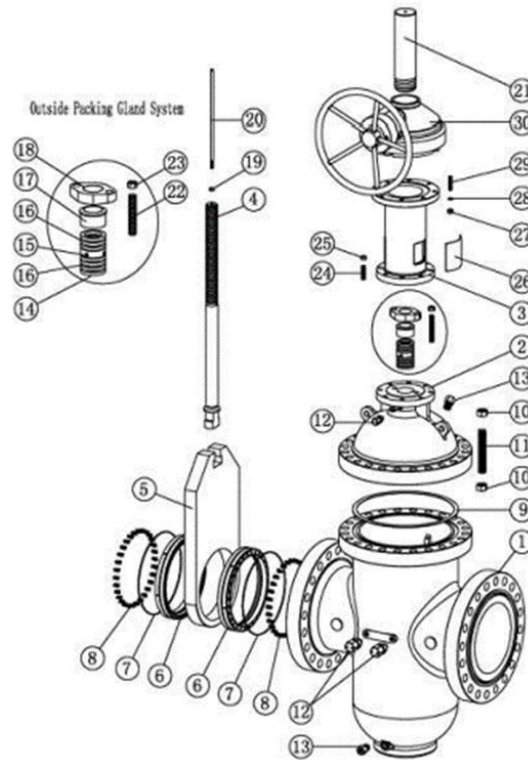


Fig. 1 Gate valve design

1 - body; 2 - cover; 3 - yoke post; 4 - spindle; 5 - gate; 6 - seat; 7 - radial seal; 8 - spring; 9 - gasket; 10 - nut; 11 - stud; 12 - lubricant fitting; 13 - drain fitting; 14 - gland seal; 15 - anti-blowout ring; 16 - secondary seal; 17 - gland bushing; 18 - gland flange; 19 - nut; 20 - spindle indicator; 21 - casing; 22 - stud; 23 - nut; 24 - bolt; 25 - nut; 26 - valve; 27 - nut; 28 - washer; 29 - stud; 30 - gearbox.

Gate valves are preferred for several reasons:

- Compact design and short overall length.
- Reliable performance under variable operating conditions.
- Minimal maintenance requirements.
- Absence of stagnant zones allows operation in slurry- or impurity-laden environments.
- Very low hydraulic resistance and laminar flow characteristics.
- Capability for quick shut-off and high tightness.

However, gate valves also suffer from wear and corrosion over prolonged use, particularly at the seat–gate interface, leading to leaks, reduced tightness, and premature failure.

3. OPERATIONAL FAILURES AND WEAR MECHANISMS

In oil field applications, gate valves experience several common malfunctions:

- Leakage between gate and seat sealing surfaces.
- Leakage between seat and body.
- Seal loss at the stem packing or bonnet-body connection.
- Fluid leakage through flange joints.

- Scoring and pitting corrosion on the spindle surface.
- Wear and cracking of sealing surfaces.
- Thread wear in the spindle sleeve.



Fig. 2 Common malfunctions during the operation of a knife gate valve at an oil field

The primary cause of valve failure is wear at the gate–seat interface, as high-velocity gas or oil flow containing mechanical impurities (e.g., H_2S , sand particles) erodes the metal surfaces. The temperature may reach up to $350^\circ C$, accelerating corrosion and abrasion processes.

The seat and bushing materials must therefore exhibit high mechanical strength, wear resistance, and corrosion stability under such conditions.

4. MATERIAL OPTIMIZATION AND TITANIUM APPLICATION

To address the wear problem, this study proposes the replacement of traditional 20X13 steel with titanium alloys, such as VT-14 or VT-6 (Fig. 3). Titanium alloys combine high corrosion resistance, low density, and excellent fatigue strength, making them ideal for harsh fluid environments.

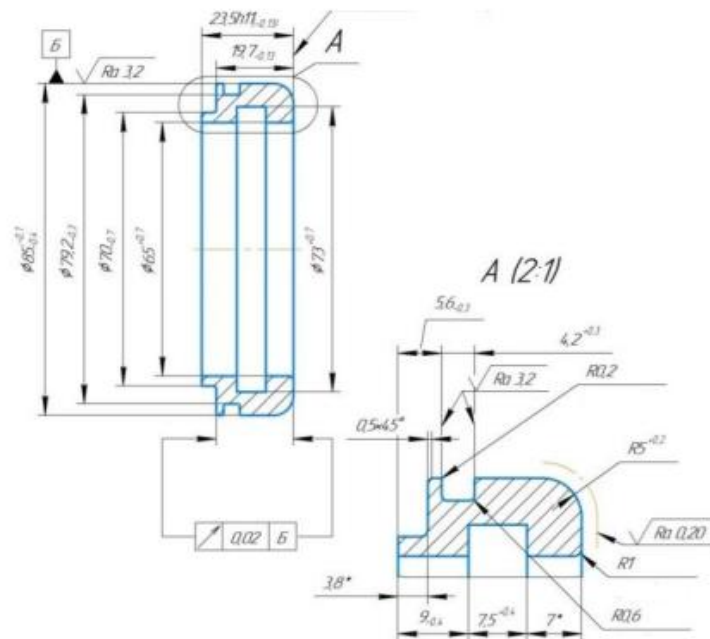


Fig. 3 Titanium seat used in valve assembly

The development was carried out within the “Production of Titanium Products for the Oil and Gas Industry” project. The resulting valve (Fig. 4) represents a **composite design** where titanium components are integrated into the high-wear zones, significantly improving longevity and sealing reliability.

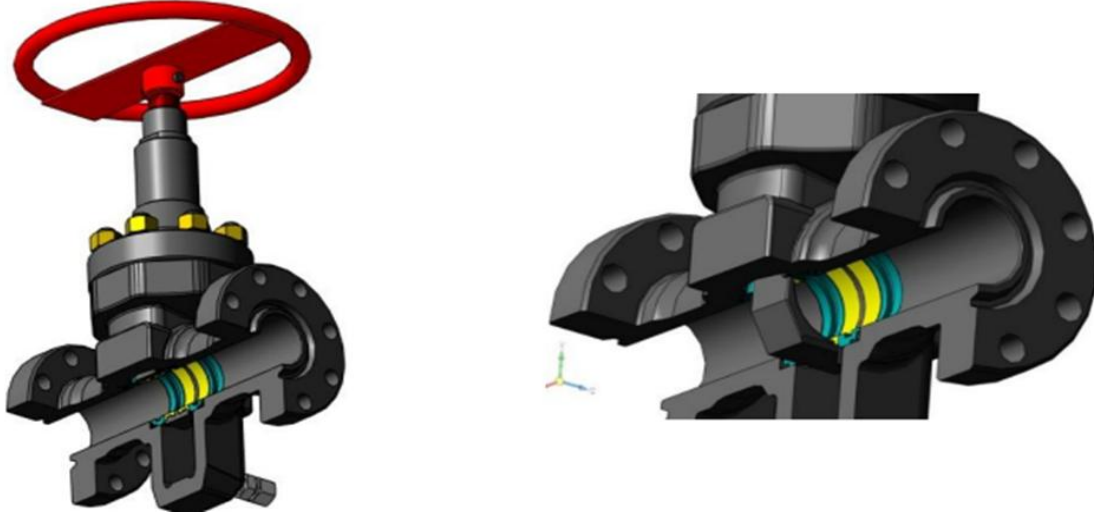


Fig. 4 Gate valve design with titanium alloy components

5. CONCLUSION

Modern shut-off valves must combine reliability, durability, and maintainability. The integration of **titanium alloy components** into gate valves enhances corrosion and wear resistance, extends operational life, and reduces maintenance costs.

Key design criteria that must be achieved include:

- Full-bore flow path ensuring minimal hydraulic resistance;
- Absence of stagnant zones;
- Capability for in-place maintenance without pipeline removal;
- Long-term tightness despite partial wear of seats.

No single conventional valve type fully satisfies all performance demands. Therefore, developing **hybrid structures** combining the strengths of various valve types — such as the titanium-integrated gate valve — is an essential step toward safer and more efficient pipeline operation.

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TECHNOLOGY FOR ASSESSING THE TECHNICAL STATE OF A HERMETIC CHILLED COMPRESSOR

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ARTICLE INFO	ABSTRACT
Article history: Received:2025-09-28 Received in revised form:2025-10-09 Accepted:2025-11-10 Available online:2026-06-30	The above technique of instrumental control of the technical state of a hermetically sealed refrigerant compressor makes it possible to perform qualitative and quantitative analysis of sound radiation from the constructive state of the device without analyzing it. Acoustic research of a domestic compressor is usually evaluated using mathematical-mechanical models, and many decisions are made on the basis of modeling. However, such an investigation is usually performed in a deterministic structure that cannot consider the epistemic uncertainties arising, for example, by the processes of colliding elements of the gas distribution mechanism. The known methods for studying the sound emission of a gas-distributing mechanism, as a source of noise, do not consider the interaction factor of the conjugate noise sources. The article compares the method of instrumental control, which makes it possible to determine the acoustic parameters on an open casing and the method of non-destructive testing of a compressor. The obtained results demonstrate reproduced and the accuracy of the study.
Keywords: technical state inspection; hermetic reciprocating compressor; hermetic refrigeration compressors; non-destructive check; acoustical radiation.	
JEL Classification: O32, L64, D24	

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1. INTRODUCTION

Reliable and safe operation of refrigeration equipment is largely associated with the analysis of these reliability parameters of the operating properties and parameters of a hermetic compressor or its individual components [1]. In this case, the control must be carried out without removing it from an operational state, i.e. without dismantling the parts of the casing that ensures the tightness of the product, the so-called non-destructive testing includes [2,3]: mean time between failures, trouble-free operating time, specified service life of the compressor. The listed compressor parameters objectively determine its main quality indicators.

Monitoring the level of technical operation of a hermetic compressor based on indicators of the actual condition of the equipment requires accurate and reliable measurement results of these indicators [4].

2. PURPOSE OF WORK

The purpose of this study is to analyze methods for assessing the technical condition of a hermetic refrigeration compressor using the example of using the acoustic method [2]. To achieve this goal, it was necessary to solve the following tasks:

1. Conduct an analytical (based on literary sources) comparison of methods for assessing the technical condition of a hermetic refrigerated compressor, in particular the acoustic method [2].
2. Conduct an analysis of the acoustic method [2] for assessing the technical condition of a hermetic cold compressor and determine approaches to solving the problem of identifying the acoustic signal generated by the processes of colliding elements of the gas distribution mechanism.

3. RESEARCH METHODOLOGY

Based on the provisions of GOST 20911-89, the concept of "technical condition" can be interpreted as a characteristic of compliance with the requirements of technical documentation. This characteristic can be expressed either in qualitative form - by designated fixed types of technical condition (serviceable, faulty, etc. GOST 27.002-89), or in quantitative form - the numerical value of the available performance resource. Characteristics can be given, compiled, found out, studied, checked. At the first stage - examine the object (characterize its technical condition). The technology of this process is quite fully presented in the following works [5, 6, 7].

The main indicators of the gas chemical complex and the determined technical level are [6]:

Efficiency coefficient K_e (COP), W/W 1.3÷1.65

Sound power level, L_a , dBA 32÷36

Mass of gas chemical complex, m kg no more than 8

Currently, the acoustic method [2] is a high-tech method for assessing the technical condition of various mechanical components, including components of a hermetic refrigerant compressor. At the same time, the method is complex, which is due to the kinematic complexity of the objects of study and the factors of acoustic interaction of vibrations from various sources of noise and vibration and the complexity of the technical equipment for conducting and analyzing the study.

The acoustic study of a compressor is usually evaluated using mathematical-mechanical models, and many decisions are made based on simulations [6, 7]. Such research is usually carried out in a deterministic framework, which cannot consider the epistemic uncertainties generated, for example, processes colliding elements of the gas distribution mechanism. Known methods for studying the process of sound radiation do not take into account the interaction factor of the associated parts of the gas distribution mechanism.

The main methods of acoustic determination of technical condition, which are used in solving problems of monitoring and functional diagnostics of refrigeration machines, are:

- measurement of vibration parameters;
- spectral analysis of the acoustic signal
- analysis of the characteristics of the rotational movement of the components of the
- compressor unit;
- peak loads;
- contour characteristic;
- shock pulse method (SPM-method Shock Pulse Method).

It is worth noting that a significant part of acoustic monitoring methods are intended for specific types of research into the acoustic characteristics of certain components and assemblies of machines.

The simplest of these methods is to measure the overall noise and vibration levels. In this case, the root mean square or peak values of the mechanical vibration parameters are measured over wide frequency ranges.

An examination by spectral analysis of noise and vibration consists of detecting periodically repeating, usually shock, processes in the operation of a machine and comparing them with possible sources of shock elastic vibrations.

Defects that affect vibration parameters, during the operation of a faulty valve mechanism, form a vibro-acoustic signal consisting of damped oscillations with different frequencies of shock origin and characterized by a significant excess of the oscillation amplitude. In most cases, by analyzing the frequency of these impacts, it is possible to identify the faulty element. A more detailed analysis is presented in [4]. Technical regulations, definitions of noise and vibration according to 17008-85, allow you to determine acoustic parameters, for example, the adjusted sound power level and vibration level, the total sound power of a hermetic compressor. The measurement result allows you to evaluate the permissible, normalized range of acoustic characteristics of the compressor and perform a comparative analysis of compressors as sources of different noise levels. The proposed methods do not allow us to identify the components of compressor noise, which in turn does not allow us to determine the influence of the design factor on the process of formation of sound vibrations of the compressor.

The structure of the proposed methodology is based on the study of structure-borne noise processes in a compressor [9]: excitation of structure-borne noise (sound radiation of impact origin); transmission of structure-borne noise (sound vibrations propagated in a solid) from the point of excitation to the point of radiation; transformation of structure-borne noise [7].

To determine the technical condition of the gas distribution mechanism of a hermetic cold compressor using a vibro-acoustic signal, it is necessary to know not only the actual, but also the corresponding reference values of elastic vibrations in compressor mechanism. The difference between the actual θ_f and the reference θ_{et} values of the diagnostic parameters is called the diagnostic symptom Δ .

$$\Delta = \theta_f - \theta_{et} \quad (1)$$

Thus, the assessment of the technical condition of the mechanism is determined by the deviation of the actual vibro-acoustic values of its parameters from their reference values.

The reference values are measured at the source of its occurrence - on the compressor housing part (2) Fig.1. Two measurements are performed: in the first measurement, the crankshaft and the movement mechanism that drives the piston in the cylinder without load.

4. DISCUSSION OF RESULTS

During the next measurement, the valve mechanism operating in backpressure mode (GOST 17008-85* not less than 10 kg/s) is brought into operation. The difference between the actual and reference values of acoustic parameters (diagnostic symptom Δ) is formed in the analyzer (6), a graphical representation is shown in Fig.1.

In accordance with the above principle, acoustic tests are carried out on a hermetic compressor, but using the non-destructive testing method, i.e., measurements are carried out with a hermetically sealed casing (in the area of the discharge pipe). The method of instrumental non-destructive testing [6], developed in collaboration with professor Naberezhnykh A.I. [1,2] allows you to determine the vibration-acoustic parameters of the gas distribution mechanism, with less labor intensity of the measurement experiment and the necessary reliability.

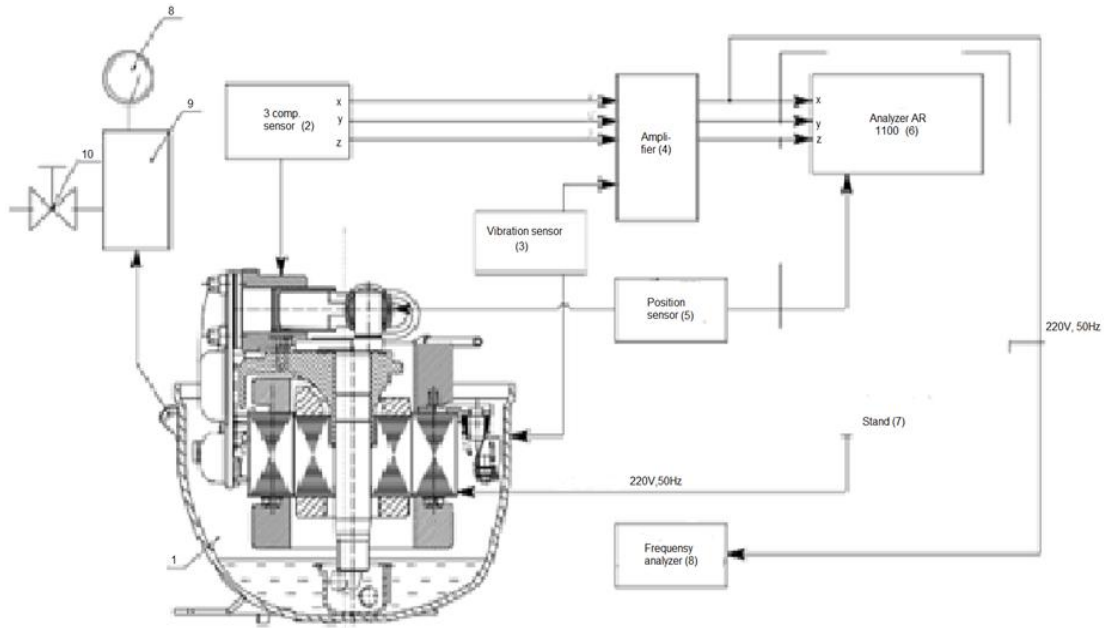


Figure 1. Functional diagram for determining the vibro-acoustic parameters of the KVO compressor in the required configuration with an open casing

1 – hermetic refrigerant compressor without a top compressor cover; 2 – three-axis vibration sensor DYTRAN 3143 M1 No. 237; 3 – Vibration sensor PCB ICP 353 B33; 4 – signal amplifier SVAN – SV 06A; 5 – Position sensor; 6 – Frequency analyzer RION-SA-27A; 6 –Yakogawa AR 1100 analyzer; 7 – stand; 8 – pressure gauge; 9 – receiver; 10 – regulating.

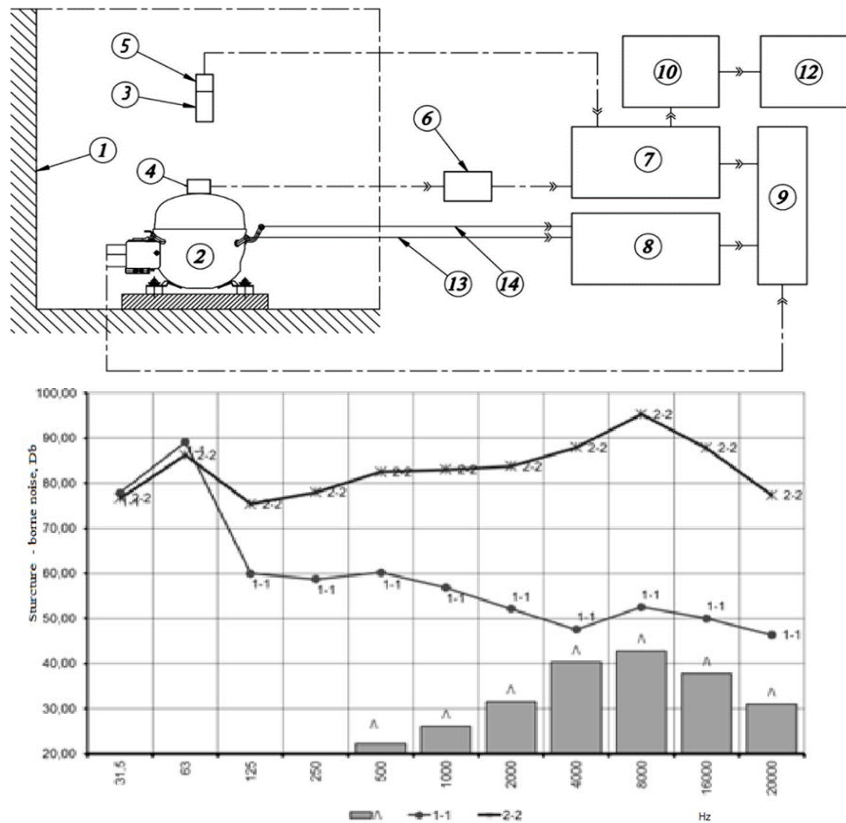


Figure 3. Vibration-acoustic characteristics of sound radiation from self-acting compressor valves:

1-1 – sound radiation of structure-borne noise of the compressor when operating in vacuum; 2-2 – acoustic radiation of structure-borne noise of a compressor under normal operating conditions, © – logarithmic difference

5. CONCLUSION

An assessment of the technical condition of the gas distribution mechanism, determined by the deviation of the actual vibro-acoustic values of its parameters from their reference values, makes it possible to estimate the magnitude of the contribution of the operation of the gas distribution mechanism to the process of sound emission of the compressor with structural detail by the components of the mechanism.

The reliability of the method is determined by the convergence of the results (correlation coefficient $K = 0.82$) with the results of the vibration-acoustic test method with a disassembled casing, with the sequential addition of noise sources.

The practical value of the presented methodology is as follows:

- assessment of the technical condition of the gas distribution mechanism, based on the analysis of sound radiation from self-acting valves without disassembling the compressor;
- the possibility of carrying out acoustic diagnostics of a self-acting gas distribution device using non-destructive testing.

Thus, the stated goal of analyzing approaches to assessing the technical condition of a hermetic refrigerated compressor using the example of using the acoustic method has been successfully completed.

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INVESTIGATION OF STRESS STATE OF A COMPOUND CIRCULAR PLATE CONNECTED BY INTERFERENCE OF A DISK WEAKENED BY AN ECCENTRIC SQUARE HOLE

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ARTICLE INFO	ABSTRACT
Article history: Received:2025-08-21 Received in revised form:2025-09-03 Accepted:2025-10-01 Available online:2026-06-30 Keywords: complex variable function; conformably mapping function; pressure; Fourier series; conjugated functions. JEL Classification: L69, O31, O33	In this article is investigated stress state of a compound circular plates. A circular plate weakened by a central circular hole in which press – fitted a circular plate weakened by an eccentrically square hole. The joint subjected on an outer contour by partially – uniformly, and by inner contour uniformly pressure. The analytic function for multiply – connected finit domains presented as two pair of power functions with unknown coefficients which must satisfy definite boundary and contact conditions. For domain is different from the circular is used conformably mapping function. By substituting the regular functions in the boundary and contact conditions obtained eight infinite system of linear algebraic equations (ISLAE) for determining unknown coefficients of the regular functions. Once the regular functions are defined by formulas Kolosov – Muskhelishvili are determined stress state of the compound circular plates.

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1. INTRODUCTION

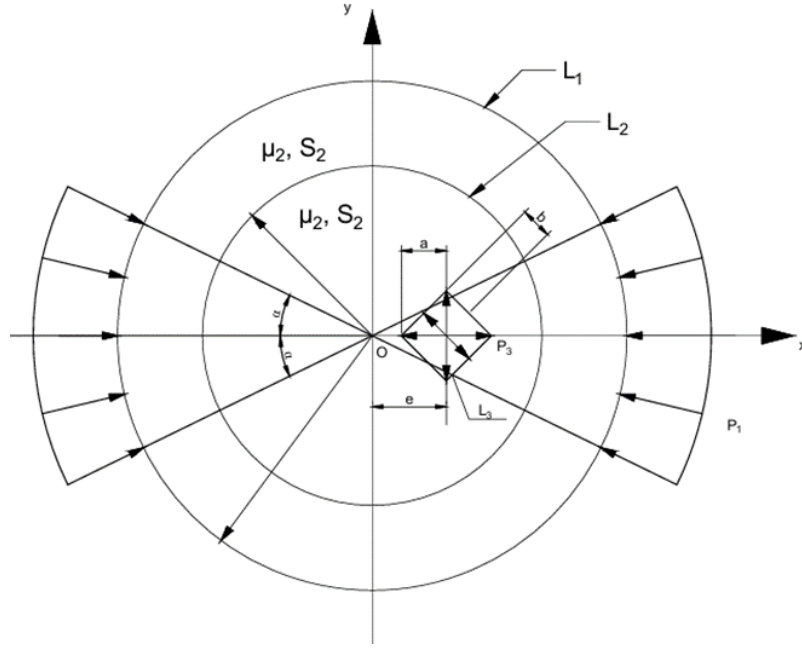
In the article is investigated stress state of compound plate. The compound plate consist of a circular plate of circular hole in which by interference a disk weakened by an eccentric square hole.

The material of plates characterizes with elastic constants μ_j and α_j ($j=1,2$) in which μ_j –the shearing modulus and ν_j is determined by the formula (Amenzade, 1996) (Bakhtiyarov, 1980).

$$\alpha_j = \frac{3-\nu_j}{1+\nu_j} \quad (1)$$

ν_j - the Poisson's coefficient.

A domain of the compound plate consist of two parts S1 and S2 (Bakhtiyarov & Ahmedov, 1985). The domain S1 bounded by concentric circles L1 and L2 and the domain S2 by concentric circle L2 and interior of a square hole (Kosmodamianskii, 1975). Besides the plate subjected on contour L1 to a partially uniformly pressure $p(t)$ and interior contour L3 by uniformly pressure $P3$ (Fig.1)



As is known, (Kuliyev, 2004), (Lavrentiev & Shabat, 1973) stress states of the compound plate are expressed by two functions $\varphi_j(z)$ and Ψ_j ($j=1,2$) of a complex variable $z = x + iy$. The functions maybe represented by the series

$$\varphi_j(z) = \sum_{k=0}^{\infty} A_k^{(j)} \left(\frac{z}{r_j}\right)^k + \sum_{k=1}^{\infty} B_k^{(j)} \left(\frac{A_j}{z - \varepsilon_j e}\right)^k \quad (2)$$

$$\Psi_j = \sum_{k=0}^{\infty} a_k^{(j)} \left(\frac{z}{r_j}\right)^k + \sum_{k=1}^{\infty} b_k \left(\frac{A_j}{z - \varepsilon_j e}\right)^k \quad (3)$$

where

$$A_j = |r_2| \text{ if } j = 1, A_j = |A| \text{ if } j = 2, E_j = \begin{cases} 0 & \text{if } j = 1 \\ 1 & \text{if } j = 2 \end{cases}$$

$A_k^{(j)}, B_k^{(j)}, a_k^{(j)}, b_k^{(j)}$ ($j = 1, 2$) - are unknown real coefficients which must be determined.

The partially – uniformly pressure subjected to exterior contour, taken as Fourier series, (Muskhelishvili, 1966), (Savin, 1968) i.e.

$$p(t) = \sum_{v=-\infty}^{\infty} \beta_v \left(\frac{t}{r_1}\right)^v, \quad (4)$$

Where,

$$\beta_v = -\frac{2P_1 r_1}{\pi(v-1)v} \sin(v-1)\alpha,$$

$$\beta_0 = -P_1 r_1 \cos\alpha,$$

$$\beta_1 = -\frac{2P_1 r_1}{\pi} \alpha,$$

$$v = \pm 1, \pm 3, \pm 5, \dots,$$

$$0 < \alpha \leq \frac{\pi}{2}$$

P_1 – intensity of external load

For domain S_2 we use Rimann's theorem (Timoshenko & Voynovskii-Krieger, 1966), (Timoshenko & Goodier, 1951) on conformal transformation:

Each simply connected domain S different from the total plane or from the plane with rejected (i.e. excluded) one point by means of an analytic function may be mapped one- to-one and conformal onto the interior or a unit disk. The precise solution of the problem on definition of a mapping function when domain is restricted by a square entails great difficulties. In order to simplify it we substitute a linear square for similar curvilinear square rounded vertices whose exterior is mapped onto exterior of a unit circle in the plane ξ by means of the function (Sherman, 1947).

$$z = A \left(\xi + \frac{m}{\xi^3} \right); A = \frac{a+b}{2}; m = \frac{1}{9} \quad (5)$$

Where a is a radius of a circle described around the square L_3 , b is a radius of circle in cribbed in the square. The linier square that we approximately substituted by a curvilinear square, nevertheless, differs from the latter, particularly, near the vertices corner points were substituted by rounded vertices.

The functions $\varphi_j(z)$ and $\Psi_j(z)$, ($j=1,2$) are regular on domains S_j ($j=1,2$) must be satisfy the following boundary and contact conditions:

$$\varphi_1(t) + t\overline{\varphi_1'(t)} + \overline{\psi_1(t)} = P(t) + C_1 \text{ on } L_1 \quad (6)$$

$$\varphi_2(t) + t\overline{\varphi_2'(t)} + \overline{\psi_2(t)} = -P_3(t - e) + C_3 \text{ on } L_3 \quad (7)$$

$$\varphi_1(t) + t\overline{\varphi_1'(t)} + \overline{\psi_1(t)} = \varphi_2(t) + t\overline{\varphi_2'(t)} + \overline{\psi_2(t)} \text{ on } L_2 \quad (8)$$

$$\frac{1}{\mu_1} (\alpha_1 \varphi_1(t) - t\overline{\varphi_1'(t)} - \overline{\psi_1(t)}) - \frac{1}{\mu_2} (\alpha_2 \mu_2 t - t\overline{\varphi_2'(t)} - \overline{\psi_2(t)}) = 2g(t) \text{ on } L_2 \quad (9)$$

in which $2g(t) = 2\delta r \left(\frac{t}{r} \right) \delta r$ – an interference of a displacement vector which is perpendicular to the line of connection; r – a radius of the disk.

t – affix point of any point of the considered L contour;

$\overline{\psi_j(t)}$ and $\overline{\varphi_j(t)}$ ($j = 1,2$) are adjoin functions for $\varphi_j(t)$ and $\psi_j(t)$ ($j = 1,2$) and $\overline{\varphi_1'(t)}$ and $\overline{\psi_1'(t)}$ their derivatives.

The boundary values of the functions $\varphi_1(z)$ and $\Psi_1(z)$ on L_1 and L_2 are expressed in the form:

$$\varphi_1(t) = \sum_{k=0}^{\infty} A_k^{(1)} \left(\frac{t}{r_1} \right)^k + \sum_{k=1}^{\infty} B_k^{(1)} \left(\frac{r_2}{r_1} \right)^k \cdot \left(\frac{r_1}{t} \right)^k \text{ on } L_1, \quad (10)$$

$$\psi_1(t) = \sum_{k=0}^{\infty} a_k^{(1)} \left(\frac{t}{r_1} \right)^k + \sum_{k=1}^{\infty} b_k^{(1)} \left(\frac{r_2}{r_1} \right)^k \cdot \left(\frac{r_1}{t} \right)^k \text{ on } L_1, \quad (11)$$

$$\varphi_1(t) = \sum_{k=0}^{\infty} A_k^{(1)} \left(\frac{r_2}{r_1} \right)^k \cdot \left(\frac{t}{r_2} \right)^k + \sum_{k=1}^{\infty} B_k^{(1)} \left(\frac{r_2}{t} \right)^k \text{ on } L_2, \quad (12)$$

$$\psi_1(t) = \sum_{k=0}^{\infty} a_k^{(1)} \left(\frac{r_2}{r_1} \right)^k \cdot \left(\frac{t}{r_2} \right)^k + \sum_{k=1}^{\infty} b_k^{(1)} \left(\frac{r_2}{t} \right)^k \text{ on } L_2, \quad (13)$$

Taking into account (10) – (13) we obtain expressions for functions $\overline{t\varphi'_1(t)}$ and $\overline{\psi_1(t)}$ on L_1 and L_2 respectively.

$$\overline{t\varphi'_1(t)} = 2A_2^{(1)} \sum_{k=1}^{\infty} \left[\left(\varepsilon_1(k)A_k^{(1)} - \varepsilon_2(k)(k-2)B_{k-2}^{(1)} \left(\frac{r_2}{r_1} \right)^{k-2} \right) \left(\frac{t}{r_1} \right)^k + (k+2)A_{k-2}^{(1)} \left(\frac{r_1}{t} \right)^k \right] \text{ on } L_1 \quad (14)$$

$$\psi_1(t) = a_0^{(1)} + \sum_{k=1}^{\infty} \left[a_k^{(1)} \left(\frac{r_1}{t} \right)^k + \left(\frac{t}{r_2} \right)^k + b_k^{(1)} \left(\frac{r_2}{r_1} \right)^k \left(\frac{t}{r_1} \right)^k \right] \text{ on } L_1 \quad (15)$$

$$\overline{t\varphi'_1(t)} = 2 \left(\frac{r_2}{r_1} \right)^2 A_2^{(1)} + \sum_{k=1}^{\infty} \left[\varepsilon_1(k) \left(\frac{r_2}{r_1} \right)^k A_k^{(1)} - \varepsilon_2(k)(k-2)B_{k-2}^{(1)} \left(\frac{t}{r_2} \right)^k + (k+2) \left(\frac{r_2}{r_1} \right)^{k+2} A_{k+2}^{(1)} \left(\frac{r_2}{t} \right)^k \right] \text{ on } L_2 \quad (16)$$

$$\psi_1(t) = a_0^{(1)} + \sum_{k=1}^{\infty} \left[a_k^{(1)} \left(\frac{r_2}{r_1} \right)^k \left(\frac{r_2}{t} \right)^k + b_k^{(1)} \left(\frac{t}{r_2} \right)^k \right] \text{ on } L_3 \quad (17)$$

The boundary value of the second member of the right portion of the $\varphi(z)$ and $\psi_1(z)$ on L_2 according to (2) and (3) is transformed by the power of variable t/r_2 , thus these functions on L_2 take a form

$$\varphi_2(t) = \sum_{k=0}^{\infty} A_k^{(2)} \left(\frac{t}{r_2} \right)^k + \sum_{k=1}^{\infty} H_k^{(2)} \left(\frac{r_2}{t} \right)^k \text{ on } L_2 \quad (18)$$

$$\psi_2(t) = \sum_{k=0}^{\infty} a_k^{(2)} \left(\frac{t}{r_2} \right)^k + \sum_{k=1}^{\infty} h_k^{(2)} \left(\frac{r_2}{t} \right)^k \text{ on } L_2 \quad (19)$$

Where,

$$H_v^{(2)} = \sum_{k=1}^v (-1)^{v-k} \cdot C_k^{v-k} \left(\frac{e}{r_2} \right)^{v-k} \left(\frac{A}{r_2} \right)^k \cdot B_k^{(2)};$$

$$h_v^{(2)} = \sum_{k=1}^v (-1)^{v-k} \cdot C_{-k}^{v-k} \left(\frac{e}{r_2} \right)^{v-k} \left(\frac{A}{r_2} \right)^k \cdot b_k.$$

Then for functions $\overline{t\varphi'_2(t)}$ and $\overline{\psi_2(t)}$ on L_2 we obtain

$$\overline{t\varphi'_2(t)} = 2A_2^{(2)} + \sum_{k=1}^{\infty} \left[\varepsilon_1(k)A_k^{(2)} - \varepsilon_2(k)A_k^{(2)} - \varepsilon_2(k)(k-2)H_{k-2}^{(2)} \left(\frac{t}{r_2} \right)^k + (k+2)A_{k-2}^{(2)} \left(\frac{r_2}{t} \right)^k \right] \text{ on } L_2 \quad (20)$$

$$\overline{\psi_2(t)} = a_0^{(2)} + \sum_{k=1}^{\infty} \left[a_k^{(2)} \left(\frac{r_2}{t} \right)^k + h_k^{(2)} \left(\frac{t}{r_2} \right)^k \right] \text{ on } L_2 \quad (21)$$

By analogy according to (2) and (3) we express functions $\varphi_2(z)$ and $\psi_2(z)$ on L_3 as an infinity series about powers of variable $t - e/A$. Then we will have

$$\varphi_2(t) = A_0^{(2)} + D_0^{(2)} + \sum_{k=1}^{\infty} \left[D_k^{(2)} \left(\frac{t-e}{A} \right)^k + B_k^{(2)} \left(\frac{A}{t-e} \right)^k \right] \quad (22)$$

$$\psi_2(t) = a_0^{(2)} + d_0^{(2)} + \sum_{k=1}^{\infty} \left[d_k^{(2)} \left(\frac{t-e}{A} \right)^k + b_k^{(2)} \left(\frac{A}{t-e} \right)^k \right] \quad (23)$$

Where,

$$D_v^2 = \sum_{K=E(v)}^{\infty} C_k^v \left(\frac{e}{r_2} \right)^k \left(\frac{A}{e} \right)^v, A_k^{(2)},$$

$$d_v = \sum_{K=E(v)}^{\infty} C_k^v \left(\frac{e}{r_2} \right)^k \left(\frac{A}{e} \right)^v, a_k^{(2)}$$

According to (22) we find,

$$\overline{\varphi_2'(t)} = \frac{D_1^{(2)}}{A} + \frac{1}{A} \left[(k+1) D_{k+1}^{(2)} \left(\frac{\bar{t}-e}{A} \right) - (k-1) B_{k-1}^{(2)} \left(\frac{A}{\bar{t}-e} \right)^k \right] \quad (24)$$

Boundary condition (7) on L_3 is transformed to a following form.

$$\varphi_2(t) + (t-e) \overline{\varphi_2'(t)} + e \overline{\varphi_2'(t)} + \overline{\psi_2(t)} = -P_3(t-e) + C_3 \quad (25)$$

By means of (5) and (22) we pass from variable $\left(\frac{t-e}{A} \right)$ to τ . Then

$$\sum_{k=1}^{\infty} D_k^{(2)} \left(\frac{t-e}{A} \right)^k = \sum_{k=1}^{\infty} D_k^{(2)} \sum_{v=-k}^{3k} * C_k^{\frac{v+k}{4}} m^{\frac{v+k}{4}} \frac{1}{\tau^v} \quad (26)$$

Taking into account that,

$$\sum_{v=-k}^{3k} * C_k^{\frac{v+k}{4}} m^{\frac{v+k}{4}} \frac{1}{\tau^v} = \sum_{v=k-4E\left(\frac{k}{4}\right)}^* C_k^{\frac{k-v}{4}} m^{\frac{k-v}{4}} \tau^v + \sum_{v=4E\left(\frac{k}{4}\right)+4-k} C_k^{\frac{v+k}{4}} m^{\frac{v+k}{4}} \frac{1}{\tau^v} \quad (27)$$

and

$$\sum_{k=1}^{\infty} \dots \sum_{v=k-4E\left(\frac{k}{4}\right)}^{k*} = \sum_{v=1}^{\infty} \dots \sum_{k=v}^{\infty} \dots,$$

$$\sum_{k=1}^{\infty} \dots \sum_{v=4-k+4E\left(\frac{k}{4}\right)}^{3k*} \dots = \sum_{v=1}^{\infty} \dots \sum_{k=\frac{v+4\alpha}{3}}^* \dots, \quad (28)$$

the formula (26) take a form

$$\sum_{k=1}^{\infty} D_k^{(2)} \left(\frac{t-e}{A} \right)^k = \sum_{v=1}^{\infty} \left[\alpha_v \cdot \tau^v + \beta_v \frac{1}{\tau^v} \right], \quad (29)$$

where,

$$\alpha_v = \sum_{k=v}^{\infty*} C_k^{\frac{k-v}{4}} m^{\frac{k-v}{4}} \cdot D_k^{(2)}; \beta_v = \sum_{k=\frac{v+4\alpha}{3}}^* C_k^{\frac{v+k}{4}} m^{\frac{v+k}{4}} \cdot D_k^{(2)} \text{ where } \alpha = 1, 2, 3 \dots$$

Symbol $E(k/4)$ signify an integrity part in $k/4$ and α – is as a smallest positive number, when $v + 4\alpha$ divided to 3 without remainder Asterisk over the series signify that in transforming to adjacent value k increased to 4.

By analogy a second expression (22) take a form

$$\sum_{k=1}^{\infty} B_k^{(2)} \left(\frac{A}{t-e} \right)^k = \sum_{v=1}^{\infty} \gamma_v \frac{1}{\tau^v} \quad (30)$$

where

$$\gamma_v = \sum_{k=1}^v {}^* C_{-k}^{\frac{v-k}{4}} m^{\frac{v-k}{4}} B_k^{(2)}$$

Therefore, the value of the function

$$\varphi_2(t) = \varphi_2^*(\tau)$$

is reduced to a form

$$\varphi_2(t) = \varphi_2^*(\tau) = A_0^{(2)} + D_0^{(2)} + \sum_{v=1}^{\infty} \left[\alpha_v \tau^v + (\beta_v + \gamma_v) \frac{1}{\tau^v} \right] \quad (31)$$

By analogy we find expression for function

$$\psi_2(t) = \psi_2^*(\tau) = a_0^{(2)} + d_0^{(2)} + \sum_{v=1}^{\infty} \left[\alpha_v^* \tau^v + (\beta_v + \gamma_v) \frac{1}{\tau^v} \right] \quad (32)$$

By means of (31) and (30) we find expression for function $\overline{\varphi_2'(t)}$

$$\overline{\varphi_2'(t)} = \frac{D_1^{(2)}}{A} + \frac{1}{A} \sum_{v=1}^{\infty} \left[(L_v - M_v) \tau^v + E_v \frac{1}{\tau^v} \right] \text{ on } L_3 \quad (33)$$

By analogy (32) for function $\psi_2(t) = \psi_2^*(\tau)$ we obtain

$$\psi_2(t) = \psi_2^*(\tau) = a_0^{(2)} + d_0^{(2)} + \sum_{v=1}^{\infty} \left[\alpha_v^* \tau^v + (\beta_v^* + \gamma_v^*) \frac{1}{\tau^v} \right] \quad (34)$$

and

$$\overline{\psi_2(t)} = \psi_2^*(\tau) = a_0^{(2)} + d_0 + \sum_{v=1}^{\infty} \left[\alpha_v^* \frac{1}{\tau^v} + (\beta_v^* + \gamma_v^*) \tau^v \right] \quad (35)$$

where

$$\alpha_v^* = \sum_{k=v}^{\infty} {}^* C_k^{\frac{k-v}{4}} m^{\frac{k-v}{4}} d_k^{(2)}; \beta_k^* = \sum_{k=\frac{v+4\alpha}{3}}^{\infty} {}^* C_k^{\frac{v+k}{4}} m^{\frac{v+k}{4}} d_k^{(2)};$$

$$\gamma_v^* = \sum_{k=1}^v {}^* C_{-k}^{\frac{v-k}{4}} m^{\frac{v-k}{4}} b_k^{(2)}; E_v = \sum_{k=v}^{\infty} {}^* (k+1) C_k^{\frac{k-v}{4}} m^{\frac{k-v}{4}} D_{k+1}^{(2)};$$

$$L_v = \sum_{k=\frac{v+4k}{3}}^{\infty} * (k+1) C_k^{\frac{k+v}{4}} m^{\frac{k+v}{4}} D_{k+1}; M_v = \sum_{k=1}^v * (k-1) C_{-k}^{\frac{k-v}{4}} m^{\frac{k-v}{4}} D_{k-1}^{(2)}$$

Taking into account (5) and (34) for functions $(t-e)\overline{\varphi_2'(t)}$ and $e \cdot \overline{\varphi_2'(\tau)}$ we obtain (Shiraliyev, 1981)

$$\begin{aligned} (t-e)\overline{\varphi_2'(t)} &= (\tau \\ &+ \frac{m}{\tau^3}) \sum_{v=1}^{\infty} \left[\left(\varepsilon_1(v) D_v^{(2)} + L - M_v \right) \tau^v + E_v \frac{1}{\tau^v} \right] \\ &= \sum_{v=1}^{\infty} \left[\varepsilon_1(v) D_v^{(2)} + \varepsilon_4(k) (L_{v-1} - M_{v-1}) + (L_{v+3} - M_{v+3}) \right] \tau^v \\ &+ \sum_{v=1}^{\infty} \left[\varepsilon_1(v) (L_{v+1} - M_{v+1}) + \varepsilon_8(v) (L_{v-1} - M_{v-1}) + E_{v+1} + \varepsilon_3(v) m D_v^{(2)} \right. \\ &\left. + \varepsilon_5(v) E_{v-3} \right] \frac{1}{\tau^v} + E_1 + L_3 - M_3 \quad (36) \end{aligned}$$

and

$$e \overline{\varphi_2'(t)} = \frac{e}{A} D_1^{(2)} + \left(\frac{e}{A} \right) \sum_{v=1}^{\infty} \left[(L_v - M_v) \tau^v + E_v \frac{1}{\tau^v} \right] \quad (37)$$

2. COMPOSITION OF LINEAR ALGEBRAIC EQUATIONS.

FORMULA FOR STRESS STATE

Substituting (10), (14), (15) and (31), (36), (37), (35) with taking into account (5) in the boundary conditions (6) and (25) respectively and substituting (12), (16) – (18), (20), (21) in contact conditions (8) and (9) respectively and comparing coefficients in the same power variables, for determining unknown coefficients $A_k^{(i)}, B_k^{(i)}, a_k^{(i)}$ and $b_k^{(i)}$ ($i = 1, 2$) ($k = 1, 2, 3 \dots \infty$) we obtain the following eight infinity system of linear algebraic equations:

$$(1 + \varepsilon_1(k)) A_k^{(1)} - \varepsilon_2(k) (k-2) \left(\frac{r_2}{r_1} \right)^{k-2} B_{k-2}^{(1)} + \left(\frac{r_2}{r_1} \right)^k b_k^{(1)} = \beta_k \quad (38)$$

$$\left(\frac{r_2}{r_1} \right)^k B_k^{(1)} + (k+2) A_{k+2}^{(1)} + a_k^{(1)} = \beta_{-k}, \quad (39)$$

$$\alpha_k + L_k - M_k + \varepsilon_4(k) (L_{k-1} - M_{k-1}) + L_{k+3} - M_{k+3} + \beta_k^* + \gamma_k^* + \varepsilon_1(k) D_k^{(2)} = \varepsilon_1(k) A \cdot P_3 \quad (40)$$

$$\begin{aligned} \beta_k + \gamma_k + \alpha_k^* + E_k + E_{k+1} + \varepsilon_5(k) E_{k-3} + \varepsilon_3(k) \cdot m D_k^{(2)} + \varepsilon_1(L_{k-1} - M_{k+1}) \\ + \varepsilon_6(k) (L_{k-1} - M_{k-1}) = \varepsilon_3(k) m A P_3, \quad (41) \end{aligned}$$

$$\begin{aligned} (1 + \varepsilon_1(k)) \left(\frac{r_2}{r_1} \right)^{k+2} A_k^{(1)} + \varepsilon_2(k) (k-2) B_{k-2}^{(1)} + b_k^{(1)} - (1 + \varepsilon_1(k)) A_k^{(2)} + \varepsilon_2(k) (k-2) H_{k-2}^{(2)} - h_k^{(2)} \\ = 0 \quad (42) \end{aligned}$$

$$B_k^{(1)} + (k+2) \left(\frac{r_2}{r_1} \right)^{k+2} A_{k+2}^{(2)} + \left(\frac{r_2}{r_1} \right)^k a_k^{(1)} - H_k - \varepsilon_2(k)(k+2)A_{k+2}^{(2)} - a_k^{(2)} = 0 \quad (43)$$

$$(\chi_1 - \varepsilon_1(k)) \left(\frac{r_2}{r_1} \right)^k A_k^{(1)} + \varepsilon_2(k)(k-2)B_{k-2}^{(1)} - b_k^{(1)} - \gamma_{12}(1 - \varepsilon_1(k))A_k^{(2)} - \varepsilon_2(k)(k-2)\gamma_{12}H_{k-2}^{(2)} + \gamma_{12}h_k^{(2)} = \varepsilon_1(k)2\gamma_1\delta r, \quad (44)$$

$$\chi_1 B_k^{(1)} - (k+2) \left(\frac{r_2}{r_1} \right)^{k+2} A_{k+2}^{(1)} - \left(\frac{r_2}{r_1} \right)^k a_k^{(1)} - \gamma_{12}H_k^{(2)} + (k+2)\gamma_{12}A_{k+2}^{(2)} + \gamma_{12}a_k^{(2)} = 0 \quad (45)$$

For free members we will have

$$A_0 + 2A_2^{(1)} + a_0 - \beta_0 = C_1, \quad (46)$$

$$A_0^{(2)} + D_0^{(2)} + (e/A)D_1^{(2)} + a_0^{(2)} + d_0^{(2)} + E_1 + L_3 - M_3 = C_3, \quad (47)$$

$$A_0^{(1)} + 2 \left(\frac{r_2}{r_1} \right)^2 A_2^{(1)} + a_0^{(1)} - A_0^{(2)} - 2A_2^{(2)} - a_0^{(2)} = 0, \quad (48)$$

$$\chi_1 A_0^{(1)} - 2 \left(\frac{r_2}{r_1} \right)^2 A_2^{(1)} - a_0 - \chi_2 \gamma_{12} A_0 + 2\gamma_{12} A_2^{(2)} + \gamma_{12} a_0^{(2)}, \quad (49)$$

where,

$$\gamma_{12} = \gamma_1 / \gamma_2$$

In order to determine $A_k^{(i)}, B_k^{(i)}, a_k^{(i)}$ and $b_k^{(i)}$ ($i = 1, 2$) on examples instead of solution infinity system of linear algebraic equations, is solved reduced system with $8N$ algebraic equations ($k=1, 2, \dots, N$) where N – is a number of restricted equations from every system, the choose which is dependent of the required correction of the solution.

Once determined approximate values of the required regular functions $\varphi_j(z)$ and $\psi_j(z)$ ($j = 1, 2$) by the following formula is determined stress components $\sigma_r^{(j)}, \sigma_\theta^{(j)}$ and $\sigma_{r\theta}^{(j)}$

$$\sigma_r^{(j)} + \sigma_\theta^{(j)} = 2[\varphi_j'(z) + \overline{\psi_j'(z)}], \quad (50)$$

$$\sigma_\theta^{(j)} - \sigma_r^{(j)} + 2i\sigma_{r\theta}^{(j)} = 2e^{2i\theta} [\bar{z}\varphi_j''(z) + \psi_j'(z)], \quad (51)$$

Where θ – an angle between external normal and axis Ox laid of from this axis.

3. Conclusion

For practical application obtained system of linear algebraic equations represented by the following compact form for easier use them in calculation

$$\sum_{i=1}^{8N} A_{ij}^{(4)} x_i = f_j^{(4)}, \quad (i, j = 1, 8N)$$

Where

$$x_n = A_n^{(1)}, x_{n+N} = B_n^{(1)}, x_{n+2N} = a_n^{(1)}, x_{n+7N} = b_n^{(1)}, x_{n+4N} = A_n^{(2)}, x_{n+5N} = B_n^{(2)}, x_{n+6N} = a_n^{(2)}, x_{n+7N} = b_{n+2}^{(2)}$$

Solution of the problem considered in the article represented by numerical examples (Table 1), on the basis which sketched stress diagrams for σ_r and σ_θ in case $P_1 = 0$ and $\frac{\sigma_r}{P_1}, \frac{\sigma_\theta}{P_1}$ in case $\delta_r = 0$ (Fig. 1 – 4). In case $P_1 = 0$, depending on the line of contact, determined radial pressure σ_r , and values compiled in Table 2 and their diagrams are shown for the case $\frac{\delta_r}{r} 10^3 = 1$ on the Fig.5

Table 1

Stress components N/m ²	Points of Z										Note
	R	0.5(R+r)	R-r	0.5(e+a+r)	e+a	e-a	-0.5r	-(R-r)	-0.5(R+r)	-R	
$-\sigma_r \cdot 10^{-6}$	0	8.54	42.73	38.79	0	0	37.09	46.71	9.12	0	$\frac{\delta_r}{r} \cdot 10^3 = 1$
$\sigma_\theta \cdot 10^{-6}$	16.27	25.01	58.93 -63	-69.23	-88.1	-90.46	-64.97	63.08 -53.9	26.24	17.24	
$-\frac{\sigma_r}{P}$	1.0	1.587	4.822	3.648	0	0	2.527	3.665	1.35	1.0	When $\delta_r = 0$ and $\alpha = 30^\circ$
$\frac{\sigma_\theta}{P}$	-0.108	-0.113	0.346 -3.892	-7.837	-23.06	-11.4	-4.986	0.947 -2.614	0.128	0.112	
Data	r/R = 0.4, e/r = 0.2, A/r = 0.36, e/A = 0.5556, m = 1/9, Materials:										
	Plate – copper ($\mu = 3.773 \cdot 10^{10} \frac{N}{m^2}, \chi = 2.008$), disk – steel ($\mu = 7.946 \cdot 10^{10} \frac{N}{m^2}, \chi = 2.2$),										

Table 2

$\frac{\delta_r}{r} \cdot 10^3$	$-\sigma_r \cdot 10^{-6} \frac{N}{m^2}$ – on the line of contact						
	$\Theta^0=0^0$	30^0	60^0	90^0	120^0	150^0	180^0
0.5	21.37	20.84	21.7	23.26	22.49	22.44	23.35
1.0	42.73	41.68	43.4	46.53	45.0	44.88	46.71
1.5	64.1	62.52	65.09	69.79	67.46	67.33	70.05
2.0	85.47	83.36	86.79	93.06	89.95	89.77	93.41
2.5	106.83	104.2	108.49	116.32	112.45	112.21	116.76
3.0	128.2	125.04	130.19	139.58	134.93	134.65	140.11

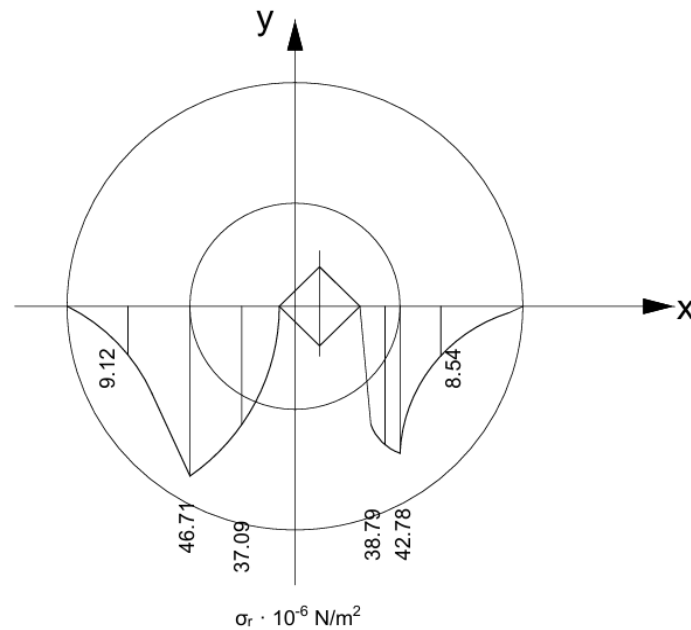


Fig. 2 Figure caption

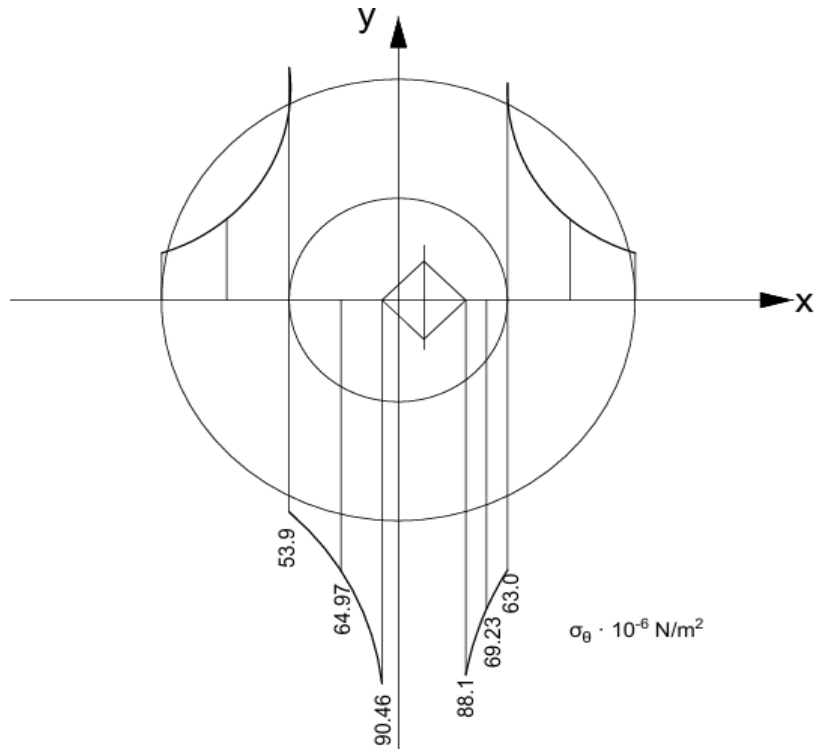


Fig. 3 Figure caption

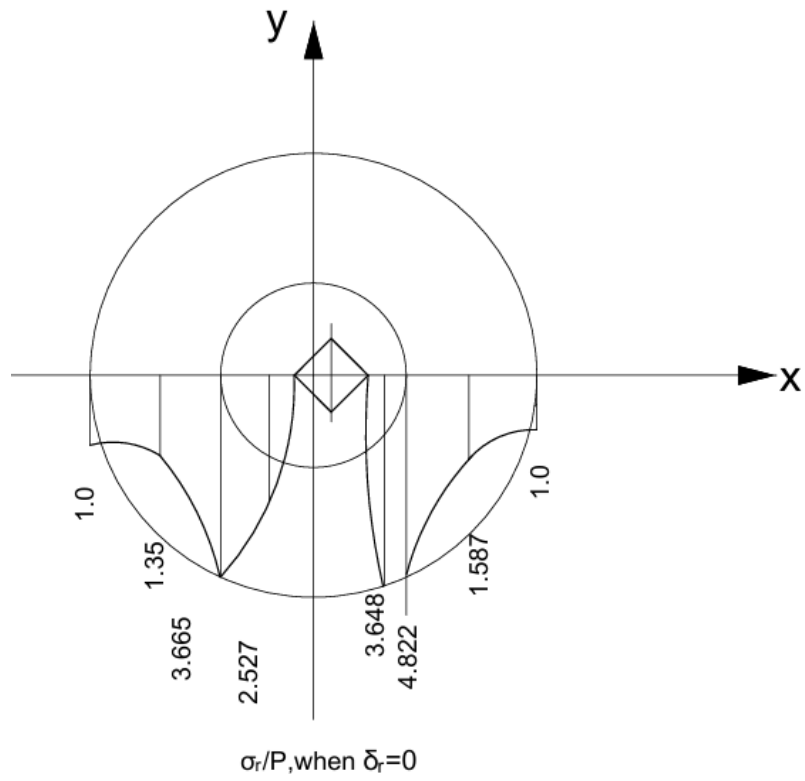


Fig. 4

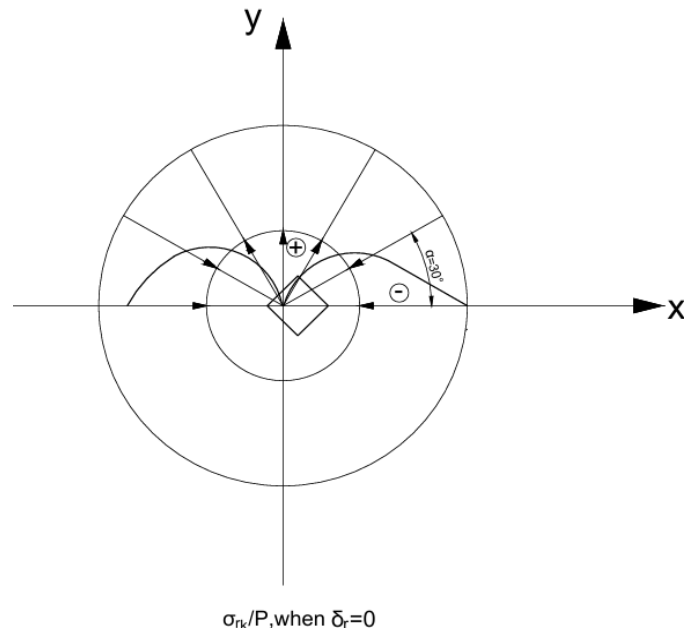


Fig. 5

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3D MODELING OF THE DEPENDENCE OF ARC OVERVOLTAGE ON THE NETWORK PHASE-TO-GROUND CAPACITANCE AND GROUND-FAULT ANGLE

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ARTICLE INFO	ABSTRACT
Article history: Received:2025-10-13 Received in revised form:2025-10-21 Accepted:2025-11-26 Available online:2026-06-30 Keywords: network isolated neutral; ground fault; overvoltage; phase capacitance; fault angle JEL Classification: L94, O32, C63	<i>In the article, the problem of determining the relationship between overvoltage arising in networks with isolated neutral as a result of non-stationary ground faults and the network phase-to-ground capacitance as well as the ground-fault angle is considered. For this purpose, by applying the least squares method, a regression equation was obtained for the dependence of the arc overvoltage factor on the network phase-to-ground capacitance and the ground-fault angle; to verify its adequacy, the statistical significance of the regression and correlation coefficients was evaluated, confidence intervals were constructed, and its 3D representation was presented. The obtained results confirm the existence of a strong correlation between these parameters and can be used for practical applications.</i>

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1. INTRODUCTION

In (Аударев, 1978), a method for testing insulation under load in networks with an isolated neutral by generating artificial overvoltages is proposed. In the proposed method, insulation testing is performed by creating an artificial ground fault in one of the network phases. However, since the ground-fault process is uncontrolled, the resulting overvoltage cannot be regulated, which is considered a drawback of the method. To overcome this limitation, a new method for testing insulation under load in networks with an isolated neutral has been proposed (Orucov & Sadiqov, 1999). According to this method, controlled artificial non-stationary ground faults are created in the network to test insulation under load. It should be noted that, during such ground faults, the magnitude of the arcing overvoltage in the network depends on the ground-fault resistance, the ground-fault angle, and the phase-to-ground capacitance of the network. Therefore, to determine the value of the test voltage, it is necessary to specify the ground-fault resistance and the ground-fault angle in advance. For this purpose, determining the dependence of the arcing overvoltage factor arising in isolated-neutral networks under non-

stationary ground faults on the ground-fault resistance, the ground-fault angle, and the phase-to-ground capacitance of the network is a relevant and important task.

2. PROBLEM STATEMENT

In general, to determine the dependences between the above-mentioned parameters, it is necessary to numerically solve a system of differential equations describing the transient process of non-stationary ground faults occurring in networks with isolated neutral, using modern computational technologies. However, the numerical solution of this problem becomes considerably complicated due to the "stiffness" of the mentioned system of differential equations; in some cases, the stability of the solution is violated and the results are distorted. Therefore, in order to overcome these difficulties, it is essential to obtain analytical expressions describing the relationships between the arc overvoltage factor (k), the ground-fault resistance (R_0), the ground-fault angle (φ), and the network phase-to-ground capacitance (C_f). For this purpose, the study considers obtaining a regression model describing the dependence of the arc overvoltage factor arising in isolated-neutral networks on the phase-to-ground capacitance of the network and the ground-fault angle.

3. PROBLEM SOLVING

For this purpose, the results of experimental investigations conducted on a low-voltage model of an isolated neutral network ($R_0 = 5 \Omega$) are utilized, as presented in Table 1 (Orucov, 1998).

Table 1
Dependence $k = f(C_f, \varphi)$

C_f, mkF	φ				
	30°	60°	90°	120°	150°
1	2,45	3,16	3,30	3,10	2,16
2	2,36	2,94	2,99	2,87	1,99
3	2,28	2,76	2,85	2,70	1,87
4	2,22	2,62	2,74	2,57	1,78
5	2,17	2,52	2,66	2,47	1,71
6	2,12	2,46	2,58	2,39	1,65
8	2,06	2,39	2,47	2,26	1,56

As can be seen from Table 1, the dependence of the arc overvoltage factor on the network phase-to-ground capacitance and the ground-fault angle can be approximated by the following regression equation (Məmmədov & Məmmədov, 2005):

$$k = \frac{a}{C_f} + b \sin \varphi + c, (1)$$

where a and b are regression coefficients, and c is the constant term of the equation.

If we introduce the substitutions $\frac{1}{C_f} = x$ and $\sin \varphi = y$ in equation (1), the regression equation can be written in the following form:

$$k = ax + by + c, (2)$$

In other words, the dependence of the arc overvoltage factor (k) on the reciprocal value of the network phase-to-ground capacitance (x) and the sine of the ground-fault angle (y) can be approximated by a linear regression equation (Table 2).

Table 2Dependence $k = f(x, y)$

x	y				
	0,500	0,866	1,000	0,866	0,500
1	2,45	3,16	3,30	3,10	2,16
0,500	2,36	2,94	2,99	2,87	1,99
0,333	2,28	2,76	2,85	2,70	1,87
0,250	2,22	2,62	2,74	2,57	1,78
0,200	2,17	2,52	2,66	2,47	1,71
0,167	2,12	2,46	2,58	2,39	1,65
0,125	2,06	2,39	2,47	2,26	1,56

The coefficients and the constant term of regression equation (2) are determined based on the least squares method (Məlikov, 2006).

According the least squares method, the values of the coefficients and the constant term of the regression equation are chosen in such a way that the sum of the squares of the errors between the approximating function and the experimental values is minimized:

$$S(a, b, c) = \sum_{i=1}^n (ax_i + by_i + c - k_i)^2 \rightarrow \min, \quad (3)$$

$S(a, b, c)$ for the function to get the smallest (minimum) value a , b and c – according to which the special derivatives given below must be equal to zero:

$$\begin{cases} \frac{\partial S(a, b, c)}{\partial a} = 0; \\ \frac{\partial S(a, b, c)}{\partial b} = 0; \\ \frac{\partial S(a, b, c)}{\partial c} = 0. \end{cases} \quad (4)$$

$S(a, b, c)$ from the function a , b and c – taking special derivatives according to (4), taking into account in the system of equations, the following system of linear equations is obtained:

$$\begin{cases} a \sum_{i=1}^n x_i^2 + b \sum_{i=1}^n x_i y_i + c \sum_{i=1}^n x_i = \sum_{i=1}^n k_i x_i; \\ a \sum_{i=1}^n x_i y_i + b \sum_{i=1}^n y_i^2 + c \sum_{i=1}^n y_i = \sum_{i=1}^n k_i y_i; \\ a \sum_{i=1}^n x_i + b \sum_{i=1}^n y_i + cn = \sum_{i=1}^n k_i. \end{cases} \quad (5)$$

To solve the system of linear equations (5) and to calculate the relevant statistical indicators, a correlation table is constructed (Table 3).

Table 3

Correlation Matrix

i	x_i	y_i	k_i	x_i^2	y_i^2	k_i^2	$x_i y_i$	$k_i x_i$	$k_i y_i$	$\left \frac{k(x_i, y_i) - k_i}{k_i} \right $	$(k(x_i, y_i) - k_i)^2$
1	1.00	0.500	2.45	1.000	0.250	6.003	0.500	2.450	1.225	0.0215	0.0028
2	1.00	0.866	3.16	1.000	0.750	9.986	0.866	3.160	2.737	0.0217	0.0047
3	1.00	1.000	3.30	1.000	1.000	10.890	1.000	3.300	3.300	0.0021	0.0000
4	1.00	0.866	3.10	1.000	0.750	9.610	0.866	3.100	2.685	0.0028	0.0001

5	1.00	0.500	2.16	1.000	0.250	4.666	0.500	2.160	1.080	0.1586	0.1174
6	0.50	0.500	2.36	0.250	0.250	5.570	0.250	1.180	1.180	0.0950	0.0503
7	0.50	0.866	2.94	0.250	0.750	8.644	0.433	1.470	2.546	0.0733	0.0465
8	0.50	1.000	2.99	0.250	1.000	8.940	0.500	1.495	2.990	0.0167	0.0025
9	0.50	0.866	2.87	0.250	0.750	8.237	0.433	1.435	2.485	0.0507	0.0212
10	0.50	0.500	1.99	0.250	0.250	3.960	0.250	0.995	0.995	0.0733	0.0213
11	0.33	0.500	2.28	0.111	0.250	5.198	0.167	0.760	1.140	0.1169	0.0710
12	0.33	0.866	2.76	0.111	0.750	7.618	0.289	0.920	2.390	0.0572	0.0249
13	0.33	1.000	2.85	0.111	1.000	8.123	0.333	0.950	2.850	0.0113	0.0010
14	0.33	0.866	2.70	0.111	0.750	7.290	0.289	0.900	2.338	0.0362	0.0096
15	0.33	0.500	1.87	0.111	0.250	3.497	0.167	0.623	0.935	0.0767	0.0206
16	0.25	0.500	2.22	0.063	0.250	4.928	0.125	0.555	1.110	0.1206	0.0716
17	0.25	0.866	2.62	0.063	0.750	6.864	0.217	0.655	2.269	0.0301	0.0062
18	0.25	1.000	2.74	0.063	1.000	7.508	0.250	0.685	2.740	0.0060	0.0003
19	0.25	0.866	2.57	0.063	0.750	6.605	0.217	0.643	2.226	0.0113	0.0008
20	0.25	0.500	1.78	0.063	0.250	3.168	0.125	0.445	0.890	0.0968	0.0297
21	0.20	0.500	2.17	0.040	0.250	4.709	0.100	0.434	1.085	0.1172	0.0647
22	0.20	0.866	2.52	0.040	0.750	6.350	0.173	0.504	2.182	0.0062	0.0002
23	0.20	1.000	2.66	0.040	1.000	7.076	0.200	0.532	2.660	0.0225	0.0036
24	0.20	0.866	2.47	0.040	0.750	6.101	0.173	0.494	2.139	0.0139	0.0012
25	0.20	0.500	1.76	0.040	0.250	3.098	0.100	0.352	0.880	0.0885	0.0242
26	0.17	0.500	2.12	0.028	0.250	4.494	0.083	0.353	1.060	0.1079	0.0523
27	0.17	0.866	2.46	0.028	0.750	6.052	0.144	0.410	2.130	0.0081	0.0004
28	0.17	1.000	2.58	0.028	1.000	6.656	0.167	0.430	2.580	0.0447	0.0133
29	0.17	0.866	2.39	0.028	0.750	5.712	0.144	0.398	2.070	0.0376	0.0081
30	0.17	0.500	1.65	0.028	0.250	2.723	0.083	0.275	0.825	0.1462	0.0582
31	0.13	0.500	2.06	0.016	0.250	4.244	0.063	0.258	1.030	0.0968	0.0397
32	0.13	0.866	2.39	0.016	0.750	5.712	0.108	0.299	2.070	0.0248	0.0035
33	0.13	1.000	2.47	0.016	1.000	6.101	0.125	0.309	2.470	0.0789	0.0379
34	0.13	0.866	2.26	0.016	0.750	5.108	0.108	0.283	1.957	0.0838	0.0358
35	0.13	0.500	1.56	0.016	0.250	2.434	0.063	0.195	0.780	0.1927	0.0904
✱	12.875	26.124	85.23	7.5351	21.0000	213.871	9.6100	33.4060	66.030	2.1487	0.9362

Based on the correlation Table 3:

$$n = 35;$$

$$\sum_{i=1}^n x_i = 12,875; \sum_{i=1}^n y_i = 26,124; \sum_{i=1}^n k_i = 85,23;$$

$$\sum_{i=1}^n x_i^2 = 7,5351; \sum_{i=1}^n y_i^2 = 21; \sum_{i=1}^n k_i^2 = 213,871;$$

$$\sum_{i=1}^n x_i y_i = 9,61; \sum_{i=1}^n k_i x_i = 33,406; \sum_{i=1}^n k_i y_i = 66,03;$$

$$\sum_{i=1}^n \left| \frac{k(x_i, y_i) - k_i}{k_i} \right| = \sum_{i=1}^n \left| \frac{ax_i + by_i + c - k_i}{k_i} \right| = 2,1487;$$

$$\sum_{i=1}^n (k(x_i, y_i) - k_i)^2 = \sum_{i=1}^n (ax_i + by_i + c - k_i)^2 = 0,9362.$$

(5) the system of linear equations is solved by one of the known methods and the regression coefficients and constant term are found, and the following values were obtained in the considered sample:

$$a = 0,73; b = 1,61; c = 0,96.$$

Thus, for non-stationary earth faults in neutral-isolated networks, the relationship (2) between the arc overvoltage factor and the reciprocal of the network's phase-to-ground capacitance and the sine of the fault inception angle can be explicitly written as:

$$k = 0,73x + 1,61y + 0,96, (6)$$

Pairwise linear correlation coefficients are defined by the following well-known expressions (Orucov, 2018):

$$\left. \begin{aligned} r_{xy} &= \frac{\overline{xy} - \bar{x} \cdot \bar{y}}{\sigma_x \sigma_y}; \\ r_{kx} &= \frac{\overline{kx} - \bar{k} \cdot \bar{x}}{\sigma_k \sigma_x}; \\ r_{ky} &= \frac{\overline{ky} - \bar{k} \cdot \bar{y}}{\sigma_k \sigma_y}. \end{aligned} \right\}, (7)$$

Here $\bar{x}$, $\bar{y}$, $\bar{k}$ – properly x , y , k average prices of quantities; $\overline{xy}$, $\overline{kx}$, $\overline{ky}$ – properly average prices of products; σ_x , σ_y , σ_k – properly x , y , k mean square deviations of their quantities are determined according to the correlation table (Table 3):

$$\begin{aligned} \bar{x} &= \frac{\sum_{i=1}^n x_i}{n} = 0,368; \quad \bar{y} = \frac{\sum_{i=1}^n y_i}{n} = 0,746; \quad \bar{k} = \frac{\sum_{i=1}^n k_i}{n} = 2,435; \\ \overline{xy} &= \frac{\sum_{i=1}^n x_i y_i}{n} = 0,275; \quad \overline{kx} = \frac{\sum_{i=1}^n k_i x_i}{n} = 0,954; \quad \overline{ky} = \frac{\sum_{i=1}^n k_i y_i}{n} = 1,887; \\ \sigma_x &= \sqrt{\frac{\sum_{i=1}^n x_i^2}{n} - \bar{x}^2} = 0,283; \quad \sigma_y = \sqrt{\frac{\sum_{i=1}^n y_i^2}{n} - \bar{y}^2} = 0,207; \quad \sigma_k = \sqrt{\frac{\sum_{i=1}^n k_i^2}{n} - \bar{k}^2} = 0,425. \end{aligned}$$

Then according to statements (7):

$$r_{xy} = 0; \quad r_{kx} = 0,488; \quad r_{ky} = 0,783.$$

The multiple correlation coefficient is determined using the following well-known expression:

$$R = \sqrt{\frac{r_{kx}^2 + r_{ky}^2 - 2r_{kx}r_{ky}r_{xy}}{1 - r_{xy}^2}} = 0,92.$$

The value of the multiple correlation coefficient being close to unity ($R = 0.92 \rightarrow 1$) indicates that the relationship between the arc overvoltage factor and the reciprocal of the phase-to-ground capacitance and the sine of the fault inception angle can be regarded as a strong linear correlation.

Verification of the adequacy of the regression equation (6) begins with the assessment of the statistical significance of the regression coefficients (Orucov, 2018).

For this purpose, Student's t -test is applied. The regression coefficients are considered statistically significant if the condition $t > t(\alpha; n - m - 1)$ is satisfied, where: t is the calculated value of Student's t -statistic; α is the significance level, $\alpha = 0.05$; n is the number of experimental observations, $n = 35$; m is the number of factors, $m = 2$; $n - m - 1$ is the number of degrees of freedom, $n - m - 1 = 35 - 2 - 1 = 32$; $t(\alpha; n - m - 1)$ is the tabulated (critical) value of Student's t -distribution,

$$t(\alpha; n-m-1) = t(0.05; 32) = 2.037.$$

The calculated values of the Student t -statistics are determined as follows. First, the standard errors of the regression coefficients are evaluated using the following well-known expressions:

$$\mu_a = \frac{\sigma_k}{\sigma_x} \cdot \frac{\sqrt{1-R^2}}{\sqrt{1-r_{xy}^2}} \cdot \frac{1}{\sqrt{n-m-1}} = 0,10404;$$

$$\mu_b = \frac{\sigma_k}{\sigma_y} \cdot \frac{\sqrt{1-R^2}}{\sqrt{1-r_{xy}^2}} \cdot \frac{1}{\sqrt{n-m-1}} = 0,14224.$$

Then, the calculated values of the Student's t -statistics corresponding to the regression coefficients are determined using the following well-known expressions:

$$t_a = \frac{a}{\mu_a} = 7,017;$$

$$t_b = \frac{b}{\mu_b} = 11,32.$$

$t_a = 7,017 > t(\alpha, n-m-1) = 2,037$ and $t_b = 11,32 > t(\alpha, n-m-1) = 2,037$ therefore, the regression coefficients a and b are considered to be statistically significant.

The confidence intervals of the regression coefficients are expressed by the following well-known inequalities (Orucov, 2018).

For the coefficient a , the confidence interval is given as:

$$a - \mu_a t(\alpha, n-m-1) < a < a + \mu_a t(\alpha, n-m-1);$$

$$a - \mu_a t(\alpha, n-m-1) = 0,73 - 0,10404 \cdot 2,037 = 0,5181;$$

$$a + \mu_a t(\alpha, n-m-1) = 0,73 + 0,10404 \cdot 2,037 = 0,9419;$$

$$0,5181 < a < 0,9419.$$

For the coefficient b , the confidence interval is given as:

$$b - \mu_b t(\alpha, n-m-1) < b < b + \mu_b t(\alpha, n-m-1);$$

$$b - \mu_b t(\alpha, n-m-1) = 1,61 - 0,14224 \cdot 2,037 = 1,3203;$$

$$b + \mu_b t(\alpha, n-m-1) = 1,61 + 0,14224 \cdot 2,037 = 1,8997;$$

$$1,3202 < b < 1,8997.$$

To verify the overall adequacy of the regression model, the significance of the multiple correlation coefficient is evaluated using Fisher's F -test (Orucov, 2018). At the significance level α , the regression equation is considered adequate if the following condition is satisfied: $F > F(\alpha; m, n-m-1)$, where m and $n-m-1$ are the degrees of freedom.

The calculated value of Fisher's F -criterion is determined from the experimental data as follows:

$$F = \frac{R^2}{1-R^2} \cdot \frac{n-m-1}{m} = 88,17.$$

The tabulated value of Fisher's F -criterion is taken from statistical tables depending on the significance level α and the degrees of freedom $(m, n - m - 1)$ (Orucov, 2018):

$$\alpha = 0,05; m = 2; n - m - 1 = 35 - 2 - 1 = 32; F(\alpha, m, n - m - 1) = F(0,05; 2; 32) = 3,29.$$

$F = 88,17 > F(\alpha, m, n - m - 1) = 3,29$ the multiple correlation coefficient and the regression equation are considered to be statistically significant.

The confidence interval of the multiple correlation coefficient is determined based on Student's t -test using the following well-known inequality (Orucov, 2018):

$$\begin{aligned} R - t(\alpha; n - m - 1) \frac{1 - R^2}{\sqrt{n}} &< R < R + t(\alpha; n - m - 1) \frac{1 - R^2}{\sqrt{n}}; \\ R - t(\alpha, n - m - 1) \frac{1 - R^2}{\sqrt{n}} &= 0,92 - 2,037 \cdot \frac{1 - 0,92^2}{\sqrt{35}} = 0,867; \\ R + t(\alpha, n - m - 1) \frac{1 - R^2}{\sqrt{n}} &= 0,92 + 2,037 \cdot \frac{1 - 0,92^2}{\sqrt{35}} = 0,973; \\ 0,867 &< R < 0,973. \end{aligned}$$

The mean relative error and the root-mean-square approximation error are determined using the following well-known expressions, respectively (Chatterjee & Simonoff, 2013):

$$\begin{aligned} \bar{\varepsilon} &= \frac{1}{n} \sum_{i=1}^n \left| \frac{k(x_i, y_i) - k_i}{k_i} \right| \cdot 100\% = \frac{1}{n} \sum_{i=1}^n \left| \frac{ax_i + by_i + c - k_i}{k_i} \right| \cdot 100\% = 6,14\%; \\ \sigma &= \sqrt{\frac{\sum_{i=1}^n (k(x_i, y_i) - k_i)^2}{n}} = \sqrt{\frac{\sum_{i=1}^n (ax_i + by_i + c - k_i)^2}{n}} = 0,164. \end{aligned}$$

The coefficient of determination is $R^2 = 0,92^2 = 0,8464$, which indicates that 84.64% of the variation in the arc overvoltage factor k is explained by the variation of the reciprocal of the network's phase-to-ground capacitance x and the sine of the fault inception angle y , while the remaining 15.36% is attributed to the influence of other unaccounted factors.

The mean elasticity coefficients are determined using the following well-known expressions (Chatterjee & Simonoff, 2013):

$$\begin{aligned} \bar{E}_x &= \frac{\partial k}{\partial x} \frac{\bar{x}}{\bar{k}} = a \frac{\bar{x}}{\bar{k}} = 0,111; \\ \bar{E}_y &= \frac{\partial k}{\partial y} \frac{\bar{y}}{\bar{k}} = b \frac{\bar{y}}{\bar{k}} = 0,493. \end{aligned}$$

From this, it follows that a 1% increase in the reciprocal of the network's phase-to-ground capacitance x leads to a 0.111% increase in the arc overvoltage factor k , while a 1% increase in the sine of the fault inception angle y causes a 0.493% increase in the arc overvoltage factor k .

Since $\bar{E}_x = 0,111 < \bar{E}_y = 0,493$, the reciprocal of the phase-to-ground capacitance x has a weaker influence on the arc overvoltage factor k compared to the sine of the fault inception angle y . The fact that the mean elasticity coefficients are less than 1% indicates that the arc overvoltage factor k is not elastic with respect to variations in the reciprocal of the phase-to-ground capacitance x and the sine of the fault inception angle y .

Thus, for non-stationary earth faults in neutral-isolated networks, the dependence (1) between the arc overvoltage factor and the network's phase-to-ground capacitance and the fault inception angle can be explicitly expressed in the following form:

$$k = \frac{0,73}{C_f} + 1,61 \sin \varphi + 0,96, \quad (8)$$

As can be seen, under non-stationary earth-fault conditions, the regression model obtained in the form of equation (8) has a simple structure and is convenient for practical implementation.

Using the OriginLab software package, a three-dimensional (3D) representation of the dependence of the arc overvoltage factor on the network's phase-to-ground capacitance and the fault inception angle was constructed based on regression equation (8) (as shown in Fig. 1).

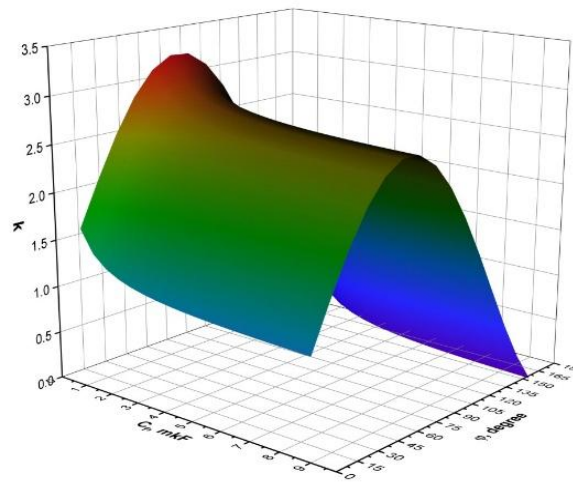


Figure 1. 3D representation of the dependence of the arc overvoltage factor on the network's phase-to-ground capacitance and the fault inception angle

4. CONCLUSION

1. As a result of controlled non-stationary ground faults, a practically easy-to-implement regression model has been obtained that relates the arcing overvoltage factor arising in networks with isolated neutral to the phase-to-ground capacitance of the network and the ground-fault angle.
2. The obtained regression model can be readily used during insulation testing under load in networks with isolated neutral.

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SOLUTION OF THE THERMOELASTIC PROBLEM IN THE DRUM-SHOE CONTACT IN A BRAKE MECHANISM

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ARTICLE INFO	ABSTRACT
Article history: Received: 2025-09-12 Received in revised form: 2025-09-24 Accepted: 2025-10-25 Available online: 2026-06-30	During braking, the temperature of drum brakes can reach 300–350°C, which changes the coefficient of friction between the lining and the drum, causing the braking torque to fluctuate. The actual values of the braking torque, as well as the deformation of the shoe and drum, depend on the contact pressure, the temperature regime of the contacting surfaces, and their strength characteristics. Knowing the actual values of the contact pressure allows one to determine the deformation values; conversely, knowing the deformation values makes it possible to determine the contact pressure. On contacting surfaces, deformation can be represented by two components: elastic and thermoelastic deformation. Given the rigidity and strength properties, geometric dimensions of the brake mechanism elements, and the forces acting on the contact surfaces, it is possible to calculate the deformation values and contact stresses during braking. This article addresses the problem of determining thermoelastic deformations in order to calculate the braking torque of drum brake mechanisms.
Keywords: braking torque; drum; brake pads; thermoelastic deformations; contact pressure.	
JEL Classification: L62, O32, C63	

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1. INTRODUCTION

Existing methods for calculating the braking torque of drum brakes, including the determination of braking torque, produce torque values that differ significantly from actual values, even when real friction coefficients are used. Current formulas for drum brake calculations are based on the conventional assumption of pressure distribution. At present, four pressure distribution laws at the drum–shoe interface are assumed a priori: uniform, sinusoidal, cosine, and square-sine. The results obtained from these approaches vary considerably. If the actual pressure on the contact surface is known, the braking force (tangential force) can be determined.

The real pressure distribution in a drum brake is typically determined experimentally. However, this process is time-consuming and costly, and accounting for the wide range of factors influencing these values is practically unfeasible.

The determination of elastic and thermoelastic deformations of the shoe and drum also requires further investigation. Existing methods for calculating the stiffness and strength of the brake drum are based on the assumption that the drum behaves as a tube or cylindrical shell subjected to uniform internal pressure. The shoe is commonly modeled either as a beam supported at its edges and loaded with a concentrated normal force or as a flat circular beam hinged at both ends.

In graphical analyses, all forces acting on the brake shoe are plotted according to a selected scale. In analytical approaches, the braking torque generated by the shoe is sometimes assumed to be infinitely large, causing the denominator in the formula to approach zero (Sakovich, N. Ye., et al., 2020). Solutions obtained under such assumptions fail to accurately reflect the real stress–strain state of the brake drum and shoes.

Any discrepancy between calculated and actual brake parameter values, regardless of direction, leads to unsatisfactory results. Therefore, the development of reliable and experimentally validated methods for drum brake calculation should be regarded as a critical issue, given its importance in the design of drum brake mechanisms.

A previously published study assumed that only elastic forces act on the contact surface (Namazov, B. F., 2024). This article extends that approach by also considering the influence of thermoelastic deformations caused by temperature increases on the friction surfaces.

2. MAIN PART

At the initial stage of calculating drum brake mechanisms, to determine the contact pressure on the drum-shoe interface, we considered the solution of an elastic problem without taking into account the temperature factor.

Since high-temperature heating and deformation occur on the friction surface of the drum and brake shoe during braking, it is crucial to study the influence of thermoelastic deformations on this contact surface in addition to elastic deformations (Chichinadze, A. V., et al., 1988). From this perspective, in the second stage of solving the problem, we study thermoelastic deformations.

To simplify the problem, we will assume that during braking we are dealing with a steady-state heat flow. Then, as is known, the temperature satisfies the equation of heat conduction theory

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \cdot \frac{\partial T}{\partial r} + \frac{1}{r^2} \cdot \frac{\partial^2 T}{\partial \theta^2} = 0 \quad (1)$$

under the following boundary conditions

$$\begin{aligned} \lambda \frac{\partial T}{\partial r} &= -q(\theta) \text{ at } \theta_0 \leq |\theta| \leq \theta_1, r = r_\delta \\ \lambda \frac{\partial T}{\partial r} + \alpha_1 (T - T_*) &= 0 \text{ at } |\theta| \leq \theta_0 \text{ and } \theta_1 \leq |\theta| \leq \pi, r = r_\delta \quad (2) \\ \lambda \frac{\partial T}{\partial r} + \alpha_2 (T - T_0) &= 0 \text{ at } r = R. \end{aligned}$$

Here α_1 – heat transfer coefficient from the inner cylindrical surface of the brake drum with temperature T_* ; α_2 – heat transfer coefficient from the outer cylindrical surface of the brake drum; $T(r, \theta)$ – temperature function. When solving the problem of heat conduction theory, it is assumed that heat exchange occurs on the outer surface of the brake drum with the external environment, which has a temperature T_0 . On the inner surface of the drum, on the contact area with the friction lining $\theta_0 \leq |\theta| \leq \theta_1$ there is a surface heat source with intensity $q(\theta)$, caused by external friction. On the rest of the drum surface $|\theta| \leq \theta_0$ and $\theta_1 \leq |\theta| \leq \pi$ heat exchange occurs with the environment having a temperature T_* (fig. 1).

The thermal conductivity coefficients of a material in the axial, circumferential, and radial directions are considered to be the same and independent of coordinates and temperature. The general solution to equation (1) is represented as

$$T(r, \theta) = 2 \operatorname{Re} F_1(z), \quad (3)$$

where

$$\begin{aligned} F_1(z) &= A \ln z + f_1(z); \\ f_1(z) &= \sum_{k=-\infty}^{\infty} b_k z^k, \end{aligned} \quad (4)$$

analytic function in a ring $r_\delta \leq |z| \leq R$; A – real constant.

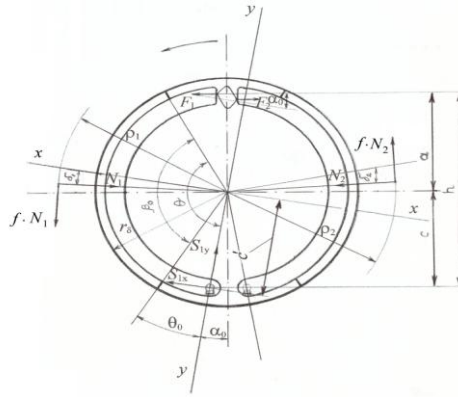


Fig. 1. Calculation diagram of the brake mechanism with fixed centers of rotation of the pads

For the function $f_1(z)$ we have boundary conditions

$$2 \operatorname{Re} [e^{i\theta} F_1'(z)] = \frac{\alpha_2}{\lambda} [T_0 - F_1(z) - \overline{F_1(z)}] \text{ at } r = R; \quad (5)$$

$$2 \operatorname{Re} [e^{i\theta} F_1'(z)] = \frac{\alpha_1}{\lambda} [T_* - F_1(z) - \overline{F_1(z)}] \text{ at } |\theta| \leq \theta_0 \text{ and } \theta_1 \leq |\theta| \leq \pi, \quad r = r_\delta; \quad (6)$$

$$2 \operatorname{Re} [e^{i\theta} F_1'(z)] = -\frac{1}{\lambda} q(\theta) \text{ at } \theta_0 \leq |\theta| \leq \theta_1, \quad r = r_\delta. \quad (7)$$

In the case where the heat transfer coefficients α_1 and α_2 large, unknown function $g(\theta)$ laid out on the inner contour $r = r_\delta$ in a Fourier series (8), (Namazov, B. F., 2024) can be replaced by the condition of constant temperature on the corresponding surfaces of the drum

$$f_1(t) + \overline{f_1(t)} + A \ln tt = T_0 \text{ at } t = \operatorname{Re}^{i\theta}; \quad (8)$$

$$f_1(t) + \overline{f_1(t)} + A \ln tt = T_* \text{ at } t = r_\delta e^{i\theta}, \quad |\theta| \leq \theta_0 \text{ and } \theta_1 \leq |\theta| \leq \pi; \quad (9)$$

$$\lambda \operatorname{Re} \left\{ e^{i\theta} \left[f_1'(z) + \frac{A}{z} \right] \right\} = -q(\theta) \text{ at } \theta_0 \leq |\theta| \leq \theta_1, \quad z = r_\delta e^{i\theta}. \quad (10)$$

Let on the contour $r = r_\delta$ ($t = r_\delta e^{i\theta}$) function $q(\theta) = \alpha_{III,1} V f g(\theta)$ expands into a complex Fourier series

$$g(\theta) = \sum_{k=-\infty}^{\infty} A'_k e^{ik\theta}. \quad (11)$$

In the designs of drum brakes of trucks, the actual conditions of heat exchange on the surface $r = r_\delta$, $|\theta| \leq \theta_0$ and $\theta_1 \leq |\theta| \leq \pi$ generally speaking, they are difficult, which is why the heat transfer coefficient is low. To simplify the problem, it can be assumed to be zero, i.e., it is assumed that on the inner surface of the drum $r = r_\delta$, $|\theta| \leq \theta_0$ and $\theta_1 \leq |\theta| \leq \pi$ there is no heat flow.

Based on the boundary conditions and relation (11) for determining $f_1(z)$ in the region occupied by the drum material, we have the following boundary conditions

$$f_1(t) + \overline{f_1(t)} + 2A \ln R = T_0, \quad t = R e^{i\theta} \quad (12)$$

$$\lambda \operatorname{Re} \left\{ e^{i\theta} \left[f_1'(\tau) + \frac{A}{\tau} \right] \right\} = -\alpha_{III,1} V f \sum_{-\infty}^{\infty} A'_k e^{ik\theta}, \quad \tau = r_\delta e^{i\theta}, \quad |\theta| \leq \theta_0 \quad \text{and} \quad \theta_1 \leq |\theta| \leq \pi. \quad (13)$$

$$\lambda \operatorname{Re} \left\{ e^{i\theta} \left[f_1'(\tau) + \frac{A}{\tau} \right] \right\} = 0, \quad \tau = r_\delta e^{i\theta}, \quad \theta_0 \leq |\theta| \leq \theta_1. \quad (14)$$

Let us remember that the coefficients A'_k are unknown to us at this time.

Substituting (5) into the boundary conditions (12 – 14) on the inner and outer surfaces of the drum and applying the procedure of the power series method, we find

$$2A \ln R + b_0 + \overline{b_0} = T_0; \quad (15)$$

$$b_k R^k + \overline{b_{-k}} R^{-k} = 0, \quad k \neq 0;$$

$$\frac{\lambda A}{2r_\delta} = -\alpha_{III,1} V f A'_0;$$

$$\frac{\lambda}{2} \left(b_1 r_\delta + \frac{A}{r_\delta} \right) = -\alpha_{III,1} V f A'_1; \quad (16)$$

$$\frac{\lambda}{2} k b_k r_\delta^{k-1} = -\alpha_{III,1} V f A'_k, \quad (k = \pm 2, \pm 3, \dots).$$

To obtain a solution to the thermoelasticity problem for a drum cross-section, we will use N.I. Muskhelishvili's dislocation analogy.

According to this analogy, the solution will consist of the solution to an "auxiliary problem," as if the body were not subjected to heating, but were subjected to a dislocation with the corresponding characteristics defined below, and a solution for the drum under the assumption of single-valued displacements. Let's consider the auxiliary problem. In our case, according to formula, we have

$$u^* + iv^* = 2 \int F_1(z) dz = 2 \left[Az \ln z - Az + b_{-1} \ln z + \sum_{k=-\infty}^{\infty} 'b_k \frac{z^{k+1}}{k+1} \right]. \quad (17)$$

The prime at the sum sign indicates that the index is excluded when summing $k = -1$.

In the case under consideration, the dislocation characteristics have the form

$$\begin{aligned}\varepsilon &= -\frac{2\pi v_1 A}{\lambda_* + \mu_1}; \quad \alpha_1^* + i\beta_1^* = 2b_{-1}; \\ \alpha_1^0 &= \frac{\pi v_1}{\lambda_* + \mu_1} \beta_1^*; \quad \beta_1^0 = \frac{\pi v_1}{\lambda_* + \mu_1} \alpha_1^*.\end{aligned}\quad (18)$$

Here λ_* and μ_1 – Lamé constants of the material. Complex “auxiliary problems” have the form (Muskhelishvili, N. I., 1977):

$$\begin{aligned}\Phi_*(z) &= B \ln z + a_0 + a_1 z + \frac{a_{-1}}{z}; \\ \Psi_*(z) &= \frac{a'_{-1}}{z} + \frac{a'_{-2}}{z^2} + \frac{a'_{-3}}{z^3}; \\ B &= \frac{\mu_1 \varepsilon}{\pi(1 + \chi_1)}; \\ a_0 &= \frac{\mu_1 \varepsilon}{2\pi(1 + \chi_1)} - \frac{\mu_1 \varepsilon (R^2 \ln R - r_\delta^2 \ln r_\delta)}{\pi(1 + \chi_1)(R^2 - r_\delta^2)}; \\ a_1 &= \frac{4\mu_1 \bar{b}_{-1}}{\pi(1 + \chi_1)(r_\delta^2 + R^2)}; \quad a_{-1} = \frac{2\mu_1 i b_{-1}}{\pi(1 + \chi_1)}; \\ a'_{-1} &= \frac{2i \bar{b}_{-1}}{\pi(1 + \chi_1)}; \quad a'_{-2} = \frac{2\mu_1 \varepsilon r_\delta^2 R^2}{\pi(1 + \chi_1)(R^2 - r_\delta^2)} \ln \frac{R}{r_\delta}; \\ a'_{-3} &= \frac{4\mu_1 i \bar{b}_{-1}}{\pi(1 + \chi_1)} \cdot \frac{R^2 r_\delta^2}{R^2 + r_\delta^2}.\end{aligned}\quad (19)$$

Here $\chi_1 = 3 - 4\nu_1$, ν_1 – Poisson's ratio of the material.

The solution to the problem for a drum with single-valued displacements was found earlier (Namazov, B. F., 2024).

For displacements in Cartesian coordinates in the case of a temperature difference, we have (Muskhelishvili, N. I., 1977):

$$u = u' + \frac{v_1 u^*}{2(\lambda_* + \mu_1)}; \quad v = v' + \frac{v_1 v^*}{2(\lambda_* + \mu_1)}. \quad (21)$$

Components u' , v' are found from the solution of the auxiliary problem (18)

$$\begin{aligned}u + iv' &= \frac{1}{2\mu_1} [\chi_1 \varphi_*(z) - z \overline{\Phi_*(z)} - \overline{\Psi_*(z)}]; \\ \varphi_*(z) &= \int \Phi_*(z) dz; \quad \psi_*(z) = \int \Psi_*(z) dz.\end{aligned}\quad (22)$$

Using relations (17), (18), (21), (22) after performing elementary, although somewhat cumbersome calculations, the radial displacement v_r we find it in the following form:

$$\begin{aligned}v_r &= v'_r + \frac{v_r v_r^*}{2(\lambda_* + \mu_1)}; \\ v'_r &= \frac{1}{2\mu_1} (c_0 + c_1 \cos \theta + c \sin \theta),\end{aligned}\quad (23)$$

$$\begin{aligned}
c_0 &= (\chi_1 - 1)Br \ln r + a_0 r + \frac{a'_2}{r}; \\
c_1 &= \chi_1 (a_0 - B) \ln r + \frac{\chi_1 \beta_1^* \mu_1 \ln r}{\pi(1 + \chi_1)} - \frac{\chi_1 \beta_1^* \mu_1 r^2}{\pi(1 + \chi_1)(r_\delta^2 + R^2)} - \frac{\mu_1 \beta_1^*}{\pi(1 + \chi_1)} + \\
&+ \frac{2\mu_1 r^2 \beta_1^*}{\pi(1 + \chi_1)(r_\delta^2 + R^2)} - \frac{\beta_1^* \ln r}{\pi(1 + \chi_1)} - \frac{\mu_1 \beta_1^* R^2 r_\delta^2}{\pi(1 + \chi_1) r^2 (R^2 + r_\delta^2)}; \\
c_2 &= \frac{\chi_1 \mu_1 r^2 \alpha_1^*}{\pi(1 + \chi_1)(r_\delta^2 + R^2)} - \frac{\chi_1 \mu_1 \alpha_1^* \ln r}{\pi(1 + \chi_1)} - \frac{\mu_1 \alpha_1^*}{\pi(1 + \chi_1)} - \\
&- \frac{2\mu_1 r^2 \alpha_1^*}{\pi(1 + \chi_1)(r_\delta^2 + R^2)} + \frac{\alpha_1^* \ln r}{\pi(1 + \chi_1)} + \frac{\mu_1 R^2 r_\delta^2 \alpha_1^*}{\pi(1 + \chi_1)(R^2 + r_\delta^2) r^2}; \\
v_r^* &= Ar \ln r - Ar + \frac{1}{2} \alpha_1^* \ln r \cos \theta - \frac{1}{2} \beta_1^* \ln r \sin \theta + \\
&+ \sum_{k=1}^{\infty} \frac{r^k + 1}{k + 1} (\alpha_k \cos k\theta - \beta_k \sin k\theta); \\
b_k &= \alpha_k + i\beta_k.
\end{aligned} \tag{25}$$

3. CONCLUSIONS AND DISCUSSIONS

The conducted research makes it possible to formulate the main findings as follows. Existing methods for calculating drum brakes, which rely on established laws of contact pressure distribution, do not accurately reflect real operating conditions and produce varying results. Therefore, a new approach has been proposed and substantiated that more closely corresponds to the physical nature of the actual loads occurring during braking.

Solving the elastic contact problem between the shoe and drum in the brake mechanism constituted the initial stage of the analysis. In this study, thermoelastic problems were subsequently addressed. To obtain a solution to the thermoelasticity problem for a drum cross-section, N. I. Muskhelishvili's dislocation analogy was employed, and formulas for calculating radial displacements were derived.

Using the obtained radial displacement values, the elastic deformation results can be refined, making it possible to determine the actual braking torque for drum-shoe brake mechanisms in heavy-duty vehicles.

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ASSESSMENT OF TRAFFIC CAPACITY IN DIFFERENT SECTIONS OF THE ROAD

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ARTICLE INFO	ABSTRACT
<i>Article history:</i> <i>Received:2025-10-09</i> <i>Received in revised form:2025-10-20</i> <i>Accepted:2025-11-20</i> <i>Available online:2026-06-30</i>	<i>The overall capacity of a road is significantly affected by the time it takes to pass through narrow spots on individual sections of the road. This time can vary from a few seconds at controlled intersections to several minutes (on steep slopes and at railway crossings). An increase in this time can lead to a sharp change in capacity and the formation of traffic jams, as well as an increase in the impact of traffic jams on the movement of vehicles. Therefore, reducing the time it takes for vehicles to pass through the road can improve traffic conditions not only in that zone, but also on the road as a whole, and increase the capacity of the road.</i>
<i>Keywords:</i> <i>road capacity; road intersection;</i> <i>roadway; traffic density;</i> <i>traffic speed.</i>	
<i>JEL Classification: R41, R42, C63</i>	

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1. INTRODUCTION

The ability of a road to carry traffic depends on many factors: road conditions (width of the carriageway, longitudinal slope, radius of curves in plan, visibility, etc.), composition of the traffic flow, availability of control devices, weather and climatic conditions, ability of vehicles to maneuver across the width of the road, psychophysiological characteristics of drivers, and design of vehicles. The variation of these factors over the course of the day, month, season, and year leads to significant changes in the capacity to allow traffic to pass. The presence of many obstacles on the road significantly affects the speed of vehicles, which leads to the formation of a large number of vehicles moving in groups and a decrease in the average speed of the entire flow [1].

The capacity of the route is significantly affected by the time spent on passing through bottlenecks on individual sections of the road. This time can vary from a few seconds at controlled intersections to several minutes (on steep slopes and at railway crossings). An increase in this time can lead to a sharp change in the capacity of traffic and the formation of traffic jams, as well as to an increase in the impact of traffic jams on the movement of vehicles. Therefore, reducing the time spent on passing bottlenecks allows improving traffic conditions not only in that zone, but also on the road as a whole, increasing the capacity of the road.

Determining the carrying capacity of a road is important not only for identifying areas requiring traffic improvement, but also for assessing the efficiency and comfort of the entire traffic flow along the route and for selecting effective traffic management methods. Any road can operate with different traffic intensities. In this case, the maximum intensity is the one that corresponds to the carrying capacity of the road. The efficiency of the transport operation of a road can be characterized both by its carrying capacity and by the intensity at which traffic on the road is most economical and optimal, given the driver's working conditions [3].

The conditions for increasing the carrying capacity of highways are:

- 1) appropriate design of plan and longitudinal profile elements that do not cause sharp changes in speed;
- 2) determination of the width of the carriageway, which allows dividing the traffic flow according to its composition (additional lanes on slopes and at level intersections) and ensures optimal loading at which movement occurs at sufficiently high speeds;
- 3) increasing the smoothness and adhesion qualities of the coating;
- 4) reconstruction of intersections at the same level or construction of intersections at different levels;
- 5) selection of regulatory means that ensure a rational traffic regime;
- 6) provision of drivers with complete information about traffic conditions along the route;
- 7) improvement of the work of the road maintenance service.

2. ASSESSMENT OF THE TRAFFIC CAPACITY OF ROAD AREAS WITHIN SMALL SETTLEMENTS

On roads within small settlements, the ability to clear traffic is low due to unorganized pedestrian movements, buildings near the road, cars and buses parked on the shoulders or in special areas, the presence of agricultural machinery in the flow, etc. The relationship between traffic speed and traffic density on roads within small settlements is expressed as follows [2]:

$$v_q = v_0 \left[1 - \left(\frac{q}{q_{max}} \right)^\beta \right]^\gamma$$

In settlements of different lengths, if the distance from the edge of the carriageway to the line of buildings is 15 - 20 m, it is accepted according to table 1.

Table 1. Parameter values

Length of road section within the boundaries of a populated area, km	v_0	β	γ
1	2	3	4
0,30 - 0,70	68,71	2,15	4,80
0,70 - 1,25	64,37	2,03	4,85
1,25 - 1,75	60,20	1,89	4,90
1,75 - 2,25	55,90	1,75	5,00

Note: small settlements include settlements with a population of up to 1,000 people, consisting of single-story buildings of a linear shape.

When determining the carrying capacity of road sections within small settlements, the maximum density of vehicle flow can be taken as the maximum density of two-lane roads. The free

movement speed of vehicles on horizontal sections of roads with a carriageway width of 7-7.5 m and shoulder width of not less than 3 m located within settlements is determined as follows [2]:

$$\vartheta_0 = 57.28 - 8.1 \cdot L + 2.3 \cdot l - 0.38 \cdot L \cdot l$$

$0.5 \leq L \leq 2.5$ km is the length of the road section within the settlement; l - is the distance from the edge of the carriageway to the line of buildings ($5 \leq l \leq 25$ km).

The greatest impact on the reduction of the capacity to pass traffic and the speed of vehicles is exerted by pedestrians crossing the roadway. Pedestrian crossings within residential areas are considered to be places that limit the capacity to pass traffic on motor roads [2]. The speed of vehicles at pedestrian crossings is determined as follows:

$$\vartheta_a = 25.4 - 0.06 \cdot N_p - 0.008 \cdot N + 0.38 \cdot \vartheta_0$$

N_p - pedestrian traffic intensity during the "peak" hour, pedestrians/hour; N - vehicle traffic intensity, vehicles/hour.

This statement is true for a pedestrian flow of 18-20% children, 50-60% adults, and 20-35% elderly.

In cases where the percentages of pedestrians change, the speed of movement of cars at pedestrian crossings is determined as follows:

$$\vartheta_a = 14.95 - 0.031 \cdot N_u - 0.041 \cdot N_B - 0.29 \cdot N_y + 0.62 \cdot \vartheta_0$$

N_u, N_B, N_y - the intensity of movement of children, adults and the elderly, respectively, in the composition of the pedestrian flow, pedestrians/hour.

The reduction in the speed of cars in residential areas is influenced by car parks and their planning features. If the park is located outside the edge of the roadway and there are acceleration lanes at the entrances to them, the speed of movement practically does not decrease. For the same reason, if there are no acceleration lanes at the entrances, the speed of movement decreases by 10-20 km/h. If there are no parks and cars are parked on the shoulder of the road, the speed of movement decreases by 20-40 km/h [4, 5]. Planda ayrırsı olan kiçik yaşayış məntəqələri hüdudundakı sahələrdə avtomobillərin sürəti aşağıdakı kimi təyin edilir:

$$\vartheta_d = \overline{\vartheta_0} - 1.29 \cdot k$$

$k = 1000/R$; R - plana ayrının radiusu ($50 > R > 100$ m).

The capacity to pass traffic on straight, horizontal road sections within small settlements is determined as follows [2]:

$$P_{ym} = (1968.8 - 487.5 \cdot L + 11.2 \cdot l + 7.5 \cdot l) \cdot k_1 \cdot k_2 \cdot k_3$$

k_1 - coefficient taking into account the impact of the pedestrian crossing; k_2 - coefficient taking into account the impact of stops in front of service points; k_3 - coefficient taking into account the radius of the curve in the plan.

Initial approximate values of the traffic capacity of road sections within the boundaries of small settlements can be determined based on figure 1.

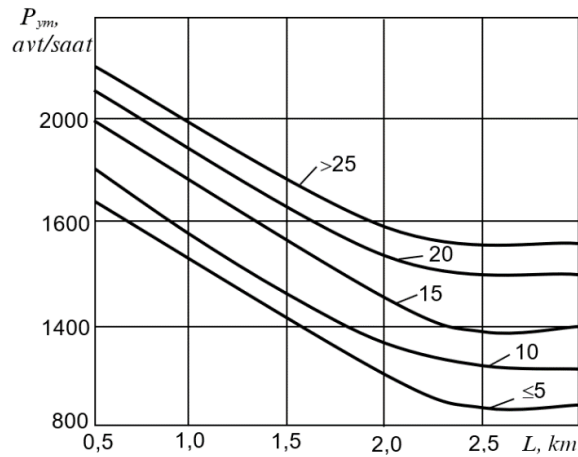


Figure 1. Dependence of the road's capacity on the length of the settlement and the distance from the roadway boundary to the building line (5-25 m).

When determining the value of the traffic capacity of the road area within the boundaries of pedestrian crossings, traffic capacity reduction coefficients are used (table 2).

Table 2. Reduction coefficients of the ability to release movement k_1

Pedestrian traffic intensity during peak hours, pedestrians/hour	Depending on the percentage of passenger cars in the stream k_1		
	100	70	50
<100	1,00	1,00	0,90
100-200	0,95	0,90	0,80
200-300	0,90	0,80	0,70
300-400	0,80	0,70	0,60

The capacity to pass traffic varies at a distance of 50 m in each direction from the pedestrian crossing. The reduction factor is equal to 1.0 at different level pedestrian crossings.

When determining the capacity of a road section in parking areas, the k_2 coefficient is used. The capacity of the traffic flow changes over a distance of 50 m in both directions from the boundary of the parking spaces in front of the service point. If a parking space is located beyond the shoulder of the roadway and is equipped with an acceleration-transition zone, $k_2 = 1.0$ is adopted; if a parking space is created by widening the shoulder, $k_2 = 0.8$; and for parking spaces without equipment, $k_2 = 0.6$ is adopted. If a parking space with equipment is located on one side of the roadway and left turns are not prohibited, the value of the k_2 coefficient should be reduced by 1.5 times [4, 5].

When determining the traffic capacity of road sections located on horizontal curves, the values of the reduction coefficients in table 3 should be used.

Table 3. The capacity of road sections located on horizontal curves to allow traffic to pass k_3

Length of settlement, km	Depending on the radius of the horizontal curve, k_3				
	100	100-250	250-450	450-600	600
0,30 - 0,70	0,81	0,89	0,95	0,96	0,97
0,70-1,25	0,84	0,92	0,97	0,98	0,98
1,25 -1,75	0,86	0,94	0,97	1,00	1,00
1,75 - 2,25	0,88	0,95	0,98	1,00	1,00
2,25 - 2,75	0,90	0,96	1,00	1,00	1,00

3. ASSESSMENT OF THE TRAFFIC CAPACITY OF BRIDGE CROSSINGS ON TWO-LANE ROADS

The capacity of bridge crossings to allow traffic depends on the design of the bridges, the condition of the carriageway and the approaches to the bridges. The actual capacity is significantly affected by the density of the flow, technical regulation means, the presence of level crossings on the approaches to the bridges, etc.

The capacity of bridges and their approaches is determined separately. The capacity of bridge approaches is evaluated as the capacity of highways.

The traffic capacity of the traffic lane of bridges located in a straight section in the plan and having a longitudinal slope of less than 10‰ is determined as follows [2]:

$$P_k = 420 + 43 \cdot Q - 2.285 \cdot L + 0.257 \cdot QL$$

Q – bridge dimensions, m ($Q7 \div Q13$); L – bridge length, m ($100 \leq L \leq 300 m$).

When determining the traffic capacity of bridges, the composition of the traffic flow should be taken into account. The traffic capacity of bridges located in a curved area in the plan and with a longitudinal slope of more than 10‰ is determined as follows:

$$P_k = P \cdot k_k$$

P – the capacity of the traffic lane taking into account the influence of curves in the plan, longitudinal slope and flow composition; k_k – the coefficient of reduction of the capacity of the bridge lane.

The values of the reduction coefficient of the traffic capacity of the bridge lane are adopted according to table 4, depending on the length and dimensions of the bridge.

Table 4. Values of the reduction coefficient of the traffic capacity of the bridge traffic lane k_k

L, m	Depending on the size of the bridge, k_k				
	Q-7	Q-8	Q-9	Q-11,5	Q-13
100	0,562	0,625	0,750	0,812	0,937
200	0,475	0,600	0,720	0,812	0,937
300	0,375	0,562	0,680	0,812	0,937

The reduction coefficients of traffic capacity for intermediate values of bridge length are taken based on nomogram 2.

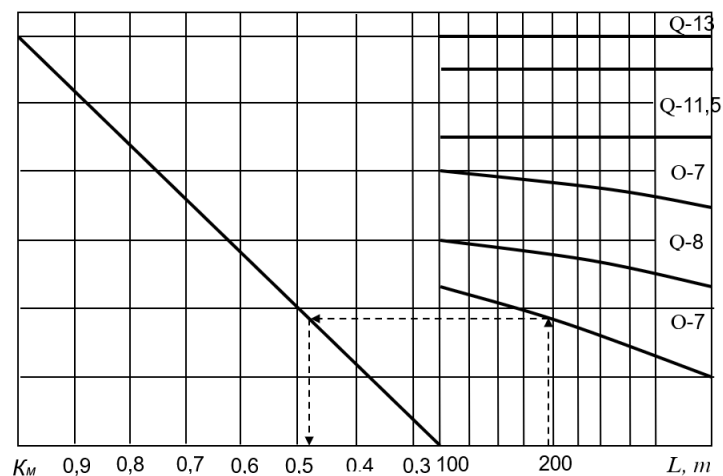


Figure 2. Nomogram for determining the reduction factor of the traffic capacity of a traffic lane on large bridges

The practical traffic carrying capacity of the traffic lane of large bridges is determined as follows [2]:

$$P_k = 0.101 \cdot v_0 \cdot q_{max}$$

P_k – traffic capacity of the traffic lane on bridges and in their zone of influence, vehicles/hour; v_0 – vehicle speed in free conditions, km/hour; q_{max} – maximum traffic density, vehicles/km.

The free movement speed of passenger cars on large bridges with a length of 100-300 m is determined as follows:

$$v_0 = 30.6 + 3.125 \cdot Q - 0.206 \cdot L + 0.01875 \cdot QL$$

4. CONCLUSION

Measures to increase the carrying capacity of the road may include:

- determining the width of the roadway, which allows the traffic flow to be divided according to its composition and ensures optimal loading at a sufficiently high speed;
- division of traffic flows into types of vehicles - this leads to the creation of unified traffic flows;
- increasing the smoothness of the pavement surface and its adhesion qualities;
- speed regulation taking into account the road load;
- selection of regulatory means ensuring a rational traffic regime;
- providing drivers with complete information about traffic conditions;
- improving the work of the road maintenance service.

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INVESTIGATION OF THE EFFICIENCY OF NEXT-GENERATION MULTISERVICE TELECOMMUNICATIONS NETWORKS BASED ON THE SDN ARCHITECTURAL CONCEPT

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ARTICLE INFO	ABSTRACT
Article history: Received:2025-08-19 Received in revised form:2025-08-30 Accepted:2025-09-29 Available online:2026-06-30	Abstract: This paper discusses the application of the Software-Defined Networking (SDN) architectural concept in the investigation of the efficiency of Next-Generation Multiservice Telecommunications Networks. The main objective is to analyze approaches that enhance the efficiency of SDN-based multiservice networks. The paper highlights the development of modern telecommunications technologies, network management issues, and the advantages of the SDN concept in addressing these challenges. It is noted that SDN networks, with software-based control and centralized management, ensure more flexible management of network resources. This approach optimizes multiservice network services, improves service quality based on traffic flow, and ensures the efficient use of network resources. The research presents sustainable analytical expressions based on models of SDN networks and their characteristics. The authors emphasize the virtualization of network resources and the provision of various services (such as voice, video, and data transmission) over a single network. These methods aim to improve service quality (QoS), automate network management, and increase productivity. In conclusion, the role of SDN network architecture in modern telecommunications networks enables the implementation of flexible and efficient approaches in providing various services. These approaches also assist network managers in controlling network traffic and optimizing resource utilization more effectively.
Keywords: multiservice; telecommunications; network; virtualization.	
JEL Classification: L96, O33, C63	

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1. INTRODUCTION

The development of modern telecommunications technologies and the increasing volume of information and service traffic transmitted over virtual communication channels require the efficient construction of multiservice communication networks that improve Quality of Service (QoS) and simplify the management of multimedia services through software-configurable network architectural concepts. It should be noted that, with the increasing load on the elements of multiservice networks, the management of telecommunications networks and information resources has become more complex [Roslyakov, A. V., (2015) Ibragimov, B. G., (2018)]. Therefore, the elements of multiservice communication networks are built on the architectural concept of next-generation networks, utilizing packet-switched systems, subscriber, network,

channel, and switching equipment, and using various signaling systems and protocols based on the new International Telecommunication Union (ITU-T) recommendations Y.3000-3499 [Loginov, S. S., (2017) Pshenichnikov, A. P. (2017)] for providing communication services.

The research results show that the architecture of commonly used multiservice communication networks is becoming increasingly complex, as telecommunication service providers are forced to support the standards of next-generation networks and protocols to offer "Triple Play services" [Stepanov, S. N. (2015)]. To address the above-mentioned challenges, a new concept is required for the architecture and operational organization of the elements of multiservice communication networks, which is recognized as the Software-Defined Network (SDN) architectural concept. An SDN network is a network architectural concept for the creation of multiservice communication networks deployed for general use with predefined parameters and program configurations [Islamov, I. J., (2021)].

Thus, the SDN network becomes programmable, using existing resources for specific applications, transforming into a more open network that ensures automatic configuration of multiservice networks according to a specific set of business applications [Islamov, I. J., (2020)]. The SDN-based programmable network allows applications to query and manipulate network services, while also providing them with the network's topology and status. One of the essential aspects of SDN is the efficient use of Network Function Virtualization (NFV), which becomes particularly important in the construction of multiservice communication networks based on next-generation network concepts.

A key feature of the SDN architecture is the separation of the data plane from the control plane through standard protocols for packet transmission. The use of the SDN architectural concept enables the establishment of modern multiservice communication networks, which is essential for creating information-communication environments and multi-operator ecosystems. This, in turn, will meet the communication service demands of public telecommunications networks, corporate and government networks, and public and private users [Islamov, I. J., (2022)].

Considering the above, this paper discusses the issue of investigating and analyzing the efficiency of the elements of multiservice communication networks based on the new SDN architecture. In general, we will review the problem formulation and the mathematical models of SDN network performance efficiency. Research shows that building multiservice communication networks based on the SDN concept is crucial for service providers to offer a wide range of multimedia services. It ensures the efficient use of information and network resources and optimizes their usage in a multi-operator environment. Considering the management of resources in multiservice communication networks based on the SDN architectural concept, the issue of resource productivity distribution and effective utilization will be formulated based on.

In this case, minimizing the average delay time during the transmission of information and service traffic and determining the effective productivity set of Software-Defined Networks (SDNs) is required. Furthermore, under the constraint of limited total productivity, OpenFlow protocols will be used. These constraints refer to the limitations associated with determining the total cost of SDN network equipment.

2. OPERATIONAL SCHEDULING OF ELEMENTS OF MULTISERVICE COMMUNICATION NETWORKS USING SDN SWITCH AND CONTROLLER BASED ON OPENFLOW PROTOCOL

Considering the importance of the interaction between software and hardware tools, terminal, network, and channel equipment, simplified mathematical models are proposed for the study of the elements of multiservice communication networks based on the concept of next-generation networks to address the above-mentioned issue.

The proposed new approach, based on the SDN network architectural concept, is described through mathematical formulas for evaluating the productivity of software and hardware tools in multiservice communication networks. This is represented by the following objective functions:

$$kf = \text{Argmax} \left\{ \prod_{k=1}^n k \right\} \quad (1)$$

Under the following constraints:

$$\eta_k \leq \eta_{k,\text{dop.}}, T_k.\text{cz} \leq T_{k,\text{cz.dop.}} \quad (2)$$

$$C_{k.\text{ap}} \leq C_{k.\text{ap.dop.}}, k = 1, \bar{n} \quad (3)$$

Here, $P_{k.\text{max}}$ represents the productivity of software and hardware tools in multiservice communication networks based on the SDN network architecture, during the transmission of the k -th packet flow, $k = 1, \bar{n}$; $C_{k.\text{ap}}$ is the cost of the equipment and software tools in SDN networks, $k = 1, \bar{n}$; $T_k.\text{cz}$ is the average dwell time of packets of the k -th traffic flow, $k = 1, \bar{n}$; $\eta_k.\text{dop.}$ is the coefficient of effective use of information, network, and channel resources in SDN networks during the transmission of the k -th packet flow.

$C_{k.\text{ap.dop.}}$, $T_{k.\text{cz.dop.}}$, $\eta_{k.\text{dop.}}$ are the accepted values for the cost of the equipment and software tools, the average delay time, and the coefficient of effective use of information and network resources in SDN networks, respectively, for the k -th packet flow, during the transmission of user and service traffic, $k = 1, \bar{n}$.

Equations (1) and (2) define the essence of the problem being considered, and based on this, a mathematical model (MM) is proposed to evaluate the performance characteristics of multiservice communication networks using the new network architectural concept (SDN) in the delivery of multimedia services.

Additionally, equations (1) and (2) describe the characteristics that allow for more accurate consideration of telecommunications processes occurring within the elements of next-generation networks using OpenFlow protocols in software and hardware tools (SDN networks).

The operational scheduling of elements in multiservice communication networks is analyzed within the framework of a model based on the SDN (Software-Defined Networks) switch and controllers using the OpenFlow protocol. Based on this model, the SDN network architecture can be used to organize the operations and interactions of network elements to ensure the provision of various services.

SDN network architecture is an approach that ensures the centralization of network management and the software-based control of network elements. This approach enables network operators to manage network resources more flexibly and regulate network services. SDN networks also provide a high level of transparency and flexibility because network elements are managed and programmed through a central controller. This allows for the network structure to be reconfigured and modified at any time.

Another advantage of the SDN model is that it enables the creation of multiple networks that are logically independent of each other and adaptable to different business applications. This supports multiservice network architectures and facilitates the delivery of various services (voice, video, data transmission, etc.) over a single network. Each service has its own packet flow and operates independently without interacting with other services on the network.

Such logical networks also improve the efficiency of business applications, as each application can use network resources according to its specific needs. Providing different services over a single network optimizes the use of network resources and increases the overall efficiency of the system. The SDN architecture not only allows network managers to control the traffic on the network but also enables them to properly assess its impact on network resources.

This approach also simplifies network operations and ensures the centralization of management operations. The SDN controller communicates with network elements to monitor their status and, when necessary, makes adjustments to the network. This helps in the more efficient management of network resources and improves the overall performance of the network.

The core idea of SDN is to separate the network management level (Control plane) from the data transmission level (Forwarding plane). All control logic and functions are moved to a separate device – the controller, which ensures centralized management based on the most optimal decisions for the network. Key benefits of SDN include rapid deployment, the flexible integration of network functions on a single server platform, and the simplicity and systematization of the management process.

According to statistical data, the application of the SDN concept significantly reduces material costs in areas such as service modernization, management, administration, and technical support.

In modern multiservice communication networks, a variety of heterogeneous traffic types are transmitted, and service-specific requirements are defined for each traffic type. Different priority heterogeneous traffic can be conditionally divided into two types:

- Useful traffic – voice, data, and video traffic;
- Service traffic – traffic used for the management and transmission of connections in multimedia services such as "Triple Play Services."

With the expansion of the spectrum of multimedia services and the increasing volume of transmitted useful and service traffic, multiservice communication networks are rapidly developing, new services are being added, and the existing service quality assurance methods often fail to meet their required tasks. Therefore, many incoming packet flows are processed with different priorities without considering network design.

The dynamic routing protocols available in network elements choose routes based solely on the efficient use of channel transmission capacity and the number of packets received. However, telecommunications operators do not take into account the actual load on the multiservice network.

Moreover, when providing multimedia services such as "Triple Play Services," most of the methods and models used for calculating multiservice networks are based on classical approaches, which do not consider the structure and self-similarity characteristics of packet-switched heterogeneous traffic, leading to improper load assessment.

To address this issue, many large operators, providers, and IT organizations are utilizing solutions based on the next-generation network concept to ensure the required Quality of Service (QoS) for useful and service traffic and to reduce the costs of providing multimedia services. A key role in this concept is the creation of a joint info communication infrastructure and multi-operator environment based on the Software-Defined Network (SDN) architectural concept for the establishment of multiservice communication networks for general use.

SDN – the Software-Defined Networking concept – is a set of methods and models that allows each subscriber to directly program, organize, manage, and control network and information resources.

The system-technical analysis of international SDN network standardization shows that these resources and capabilities help ensure the availability of "Triple Play Services" type multimedia services, increase the requirements for network management and transmission mechanisms, and escalate the need for continuous improvement of these networks.

Considering the above, SDN, which enables the dynamic and efficient distribution and management of information and network resources, will set a new standard in the creation of next-generation network systems due to the advantages of the network architectural concept.

Research has shown that the key advantages of SDN networks in creating effective elements for multiservice communication networks for general use are as follows:

1. **Centralized Management:** SDN networks ensure centralized management of information and network resources, facilitating easier and more efficient network operations in multiservice communication networks and multi-operator environments.
2. **Automation and Reduced Complexity:** Automation of service and management processes reduces the complexity of multiservice networks, simplifying the work of network managers and providing more flexible management capabilities.
3. **Increased Transmission and Service Speed:** Increasing the speed of services and data transmission in a multiservice environment ensures higher quality service for users and improves the overall performance of the network.
4. **Reliability and Security:** The reliability and security of networks for general use are enhanced through centralized management and the optimization of network resources offered by the SDN architecture. This minimizes network failures and security threats.
5. **Policy Compliance:** Ensuring compliance with management policies regarding subscriber and network access improves the quality of network services and provides more stable and reliable connectivity for users.

6. Quality of "Triple Play Services": SDN ensures the improvement of quality parameters for various multimedia services (Triple Play Services: voice, video, and internet), thereby delivering higher service quality to users.
7. Guaranteed Quality of Service (QoS): SDN networks enhance the guaranteed Quality of Service (QoS) for both useful and service traffic, as well as engineering traffic, ensuring more efficient use of network resources and high-quality service delivery.
8. Increased Speed of Innovation Implementation: The flexibility and programmability of SDN enable faster implementation of new technologies and innovations, helping network operators remain competitive in the market.
9. Narrow-Scope Network Connection Management: The narrow-scope management of SDN network elements enables more efficient use of network resources and more precise control over services.
10. Improved Quality of Experience (QoE): The services offered by SDN networks enhance the users' Quality of Experience (QoE), increasing their positive experiences and thereby improving customer satisfaction.

In light of the above, it is necessary to explore new solutions for analyzing the indicators of SDN networks—models and research methods for massive service systems reflecting the infrastructure of next-generation multiservice communication networks and describing telecommunication processes.

Now, let us examine an example of a typical segment of SDN networks using a switch and controller operating based on the OpenFlow protocol. Figure 1 presents the operational diagram of an element of multiservice communication networks using an SDN switch and controller operating based on the OpenFlow protocol.

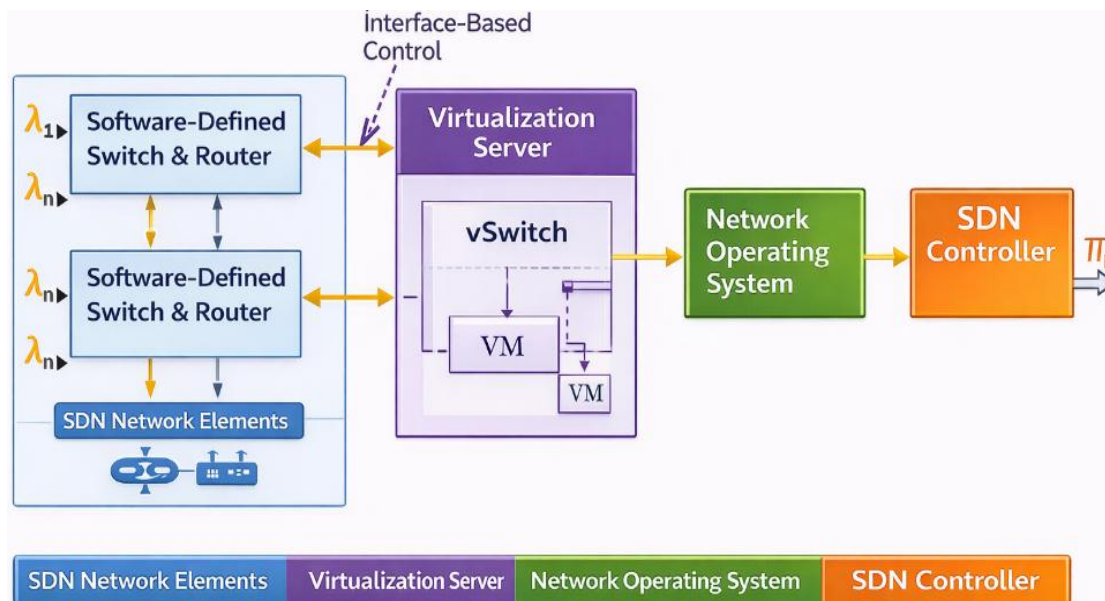


Figure 1. Operational Diagram of Multiservice Communication Networks Using SDN Switch and Controller Based on OpenFlow Protocol

As shown in the figure, the network elements primarily consist of SDN network elements, network function virtualization servers, network operating systems, and the SDN controller.

3. INVESTIGATION AND ANALYSIS OF THE INDICATORS OF MULTISERVICE NETWORK ELEMENTS FOR GENERAL USE BASED ON THE SDN NETWORK CONCEPT

In this section, it is assumed that the traffic flow entering each SDN network switch consists of useful and service data packets with specific headers. The network controller determines the set of forwarding rules corresponding to the incoming header and sends these rules back to the switch. As a result, the packets are queued, and the switch processes the traffic packets with different priorities in sequence.

This approach is designed to optimize the management and transmission of data over the network. In SDN networks, the traffic packets received by switches are routed according to predefined rules, and each packet is processed based on a specific priority. This process allows for the more accurate and efficient management of traffic flows and enables more efficient use of network resources. The quality management of the analyzed indicators and the provision of service quality (QoS) play a significant role in multiservice methods and teletraffic theory models. These methods ensure the optimal distribution of traffic flows for different types of services over the network. For instance, optimal distribution between services with different priorities and useful data helps efficiently manage the network load.

The analytical methods and models are also designed to improve network performance and user experience. These approaches enhance the management of network traffic and the interaction between services, ensuring that SDN networks operate faster and more reliably. As a result, the application of these approaches leads to an increase in network service quality and positive outcomes for service providers.

Research in this field shows that the SDN network architecture increases the flexibility and efficiency of multiservice communication network management, which leads to a more optimized use of network resources, improved service quality, and enhanced overall system performance.

For the communication channel, we accept the speed as C_k in the SDN network element. Therefore, the greatest common divisor (GCD) of these speeds is referred to as the unit of the channel resource, and it is expressed as follows:

$$E = \text{GCD}(v_k, C_k) = \text{GCD}(w(v_1, v_2, \dots, v_n), C_k) \quad (4)$$

From the analysis, it is evident that the communication channel speed of the network element, v_k , defines the channel units, and consequently, the service speed requirement for the k -th packet flow is determined as follows:

$$v_k = \frac{C_k}{\text{GCD}(v_1, v_2, \dots, v_n)}, b_k = \frac{v_k}{E}, k = 1, n \quad (5)$$

Here, b_k is the number of channel resource units required to serve a packet for the k -th traffic flow.

Based on equations (3) and (4), the quality indicators of joint service for useful and service traffic can be determined:

- Considering the probability $p(i)$ that the service system is in the i -th state, the fraction of lost k -th packet flows can be determined as follows:

$$p_{k,n} = \sum_{i=m}^{C_k} p(i), m = C_k - b_k + 1 \quad (6)$$

Where C_k is the operational speed of the SDN network element, expressed in units of channel resources.

It should be noted that the set of states in the SDN network element is finite, and it consists of vectors $i_k = i_1, i_2, \dots, i_n$, where $k = 1, \bar{n}$, and agrees with the following inequality:

$$\sum_{k=1}^n i_k \cdot b_k \leq C_k, k = 1, \bar{n} \quad (7)$$

Expression (7) defines the coefficient of effective use of resources in the SDN network element during the transmission of the k -th packet flow, and it represents the total number of channel resources required to serve all traffic flow packets in the network element.

For the normal operation of the SDN network element, when there is no bounded increase in the queue, expression (7) will be as follows:

$$\eta_{r.} = k = 1n [Ld(Nk \cdot Ck) - 1 \cdot ik] \leq \eta_{r.dop.}, k = 1, \bar{n} \quad (8)$$

Where L_d is the length of the transmitted packet of the traffic flow; N_k is the number of communication channels used for packet transmission in the SDN network element.

The fulfillment of the last condition allows for the determination of the channel resource reserve of the SDN network element:

$$R(N_k) = 1 - \eta_{r.}(N_k), k = 1, \bar{n} \quad (9)$$

Equations (4), ..., (8) determine the productivity indicators of the PKS network element and the quality (QoS) indicators of the joint service of traffic flows using software-controlled switches and controllers.

Additionally, the relationships obtained based on the mathematical model (RM) from equations (5), (6), (7), and (9) allow for a more accurate assessment of the effectiveness of resource use and management in SDN network elements.

- In the system, when the values of $p(i)$ are known, we will determine the average value of the channel resources used to serve the k -th packet flow, which is one of the important indicators of QoS service quality, and it is given by:

$$M[\eta_k] = \rho_k \cdot (1 - p_{k,n}) \cdot b_k, k = 1, \bar{n} \quad (10)$$

Where ρ_k is the intensity of the useful and service traffic load for the k -th packet flow.

For the evaluation and management of channel resources, in equations (2.10) and (2.10), the probability $p(i)$, which plays a special role, is the normalized probability, and its determination is related to the following recurrent relations:

$$p(i) = \frac{1}{i} \cdot k = 1, n \cdot p \cdot b_k(i - b_k) \cdot \rho_k \cdot I[(i - b_k) \geq 0], k = 1, \bar{n} \quad (11)$$

Expression (2.11) characterizes the probability of the service system being in the i -th state. Furthermore, the condition in (10) is also satisfied for non-normalized values.

Thus, based on the derived analytical expressions, the probabilistic-time characteristics of the elements of next-generation multiservice communication networks can be determined.

The productivity indicators of the SDN network element and next-generation network elements, as well as the effective dynamic distribution and management of information and network resources, have been analyzed.

4. CONCLUSION

1. The SDN (Software-Defined Networking) architecture centralizes network management, ensuring more flexible and efficient use of resources. This approach simplifies the operations of telecommunications networks and improves service quality (QoS), thereby providing users with more stable and reliable connections.
2. SDN networks enhance the quality of various multimedia services, such as "Triple Play Services," in multiservice communication networks. The centralized management provided by SDN ensures the optimal use of network resources, leading to improved network performance and service quality.
3. The flexibility and programmability of SDN enable the faster implementation of new technologies. This allows network operators to remain competitive in the market and achieve positive results in their services through the rapid application of innovations.

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MULTI-METHODOLOGICAL OPTIMIZATION IN MECHANICAL SYSTEMS: VISUAL-GEOMETRIC APPROACHES AND INDUSTRIAL APPLICATIONS

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ARTICLE INFO	ABSTRACT
Article history: Received:2025-10-06 Received in revised form:2025-10-28 Accepted:2025-11-30 Available online:2026-06-30 Keywords: Mechanical Systems; Geometric Optimization; Industrial Engineering; Electric Vehicles; Mathematical Modeling. JEL Classification: O32, C61, L60	This research investigates a robust multi-methodological optimization framework designed to solve complex challenges frequently encountered in modern mechanical and industrial engineering. While traditional derivative-based calculus remains the standard analytical tool for finding extreme values, it often presents significant computational hurdles and algebraic complexity, especially when dealing with non-linear models and multi-variable constraints. We propose a transition toward a "visual-geometric" approach that leverages physical intuition, geometric transformations, and fundamental algebraic inequalities, such as the AM-GM principle, to streamline the optimization process. The study provides detailed case studies on the optimization of radical sums, trapezoidal structural supports, and shortest-path logistics using parallel translation and reflection techniques. The practical utility and efficiency of these "short-cut" strategies are further demonstrated through critical industrial applications, including permanent magnet synchronous motor (PMSM) efficiency, electric vehicle (EV) battery pack configuration, and power infrastructure routing. Comparative results indicate that the proposed visual methodology significantly reduces cognitive load and analytical time without compromising numerical precision, offering a vital pedagogical and professional tool for enhancing engineering design performance.

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1. INTRODUCTION

Traditionally, engineering optimization problems are approached through differential calculus, involving the derivation of objective functions [5]. However, in non-linear systems, this can obscure physical intuition. This paper introduces a visual-geometric framework as a complementary tool, demonstrating how "geometric shortcuts" provide immediate, exact solutions.

2. MATHEMATICAL FOUNDATIONS AND METHODOLOGY

1. **Geometric Synthesis:** Representing algebraic terms as physical dimensions transforms complex minimization into "shortest path" problems [3].
2. **The AM-GM Principle:** Provides a powerful boundary for optimization without differentiation, effective in structural load calculations [1], [6].

3. CASE STUDY A: OPTIMIZATION OF RADICAL SUMS

an industrial routing problem where the total length of a three-segment system must be minimized. The objective function is given as:

$$f(x, y, z) = \sqrt{x^2 + 1} + \sqrt{y^2 + 1} + \sqrt{z^2 + 1} \quad (1)$$

$$\text{Subject to the constraint: } x + y + z = 4 \quad (2)$$

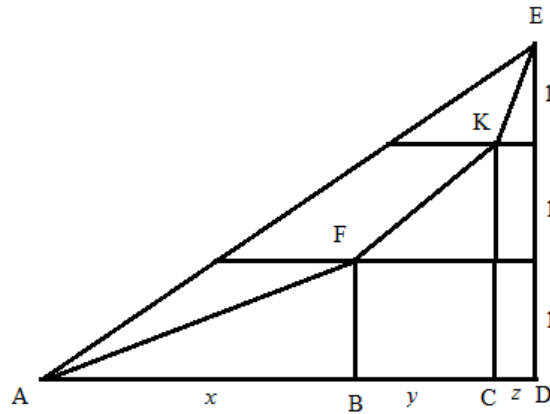


Fig. 1. Geometric synthesis of radical sums.

Visual-Geometric approach: Each radical term is viewed as the hypotenuse of a right triangle with base x and height 1. By placing these three triangles adjacently, the minimum total length is the straight line connecting the start and end points. The resulting "grand triangle" has a total base of 4 and a total height of 3 as $(1+1+1)$. Applying the Pythagorean theorem:

$$L_{min} = \sqrt{4^2 + 3^2} = 5 \quad (3) [1]$$

4. CASE STUDY B: STRUCTURAL CONSTRAINTS IN TRAPEZOIDAL ELEMENTS

In mechanical design, trapezoidal cross-sections are frequently used for structural stability. A specific problem involves a trapezoid with a fixed area $S = 10$, where the goal is to maximize the length of the shorter diagonal d for a given height h . The relationship is modeled as:

$$d = \sqrt{\left(\frac{S}{h}\right)^2 + h^2} \quad (4)$$

By applying the AM-GM inequality to the squared terms:

$$d^2 = \left(\frac{S}{h}\right)^2 + h^2 \geq 2\sqrt{\frac{S}{h} \cdot h} = 2S \quad (5)$$

For $S = 10$, $d^2 \geq 20$, thus: $d \geq 2\sqrt{5}$ [1].

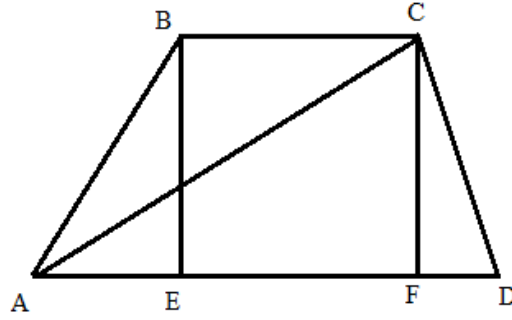


Fig. 2. Structural optimization of diagonal supports in a trapezoidal frame.

5. CASE STUDY C: OPTIMIZATION OF PATHS WITH FIXED CONSTRAINTS

Another critical problem in industrial logistics and infrastructure design involves finding the shortest path between two points, A and B, that must include a fixed-length segment MN along a specific axis.

Problem Definition:

Given points $A(0, 4)$ and $B(12, 8)$, find the minimum path $AM + MN + NB$, where MN is a segment of fixed length $d=3$ moving along the x-axis.

Visual-Geometric Methodology:

To solve this without complex optimization algorithms, we employ a **parallel translation** followed by a **reflection**:

1. **Translation:** We shift point A horizontally by the length of segment MN ($d=3$) to a new position $A'(3, 4)$.
2. **Shortest Path:** The problem now reduces to finding the shortest distance between A' and B.
3. **Calculation:** Using the distance formula between $A'(3, 4)$ and $B(12, 8)$:

$$A'B = \sqrt{(12 - 3)^2 + (8 - 4)^2} = \sqrt{97} \quad (6).$$

4. **Total Distance:** The total minimum length is $L_{min} = \sqrt{97} + 3$ (7).

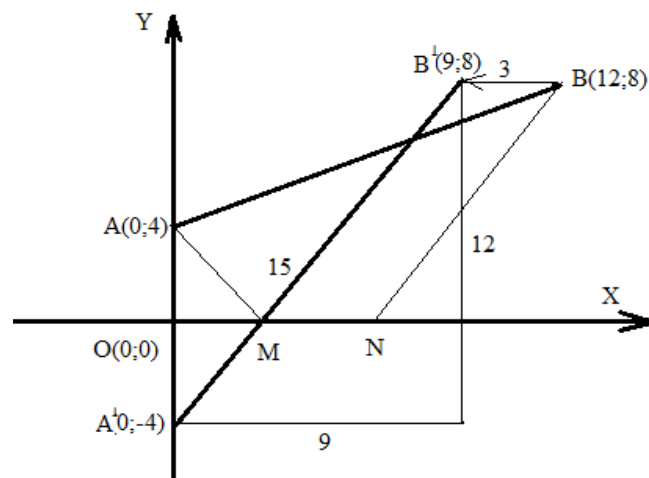


Fig. 3. Shortest Path with MN segment.

6. INDUSTRIAL APPLICATIONS

- **EV Motor Efficiency:** Visual-geometric methods identify MTPA points quickly.
- **Battery Pack Configuration:** Optimization of diagonal supports ensures mechanical integrity in EV chassis.
- **Grid Infrastructure:** Shortest path algorithms are critical for minimizing copper weight in distribution networks [2], [5].

7. RESULTS AND CONCLUSION

This research demonstrates that integrating visual-geometric strategies alongside traditional calculus significantly enhances the analytical capabilities in mechanical and industrial engineering. By reducing the cognitive load required for purely algebraic manipulation, these methods allow engineers to maintain a robust connection with the physical and geometric realities of their designs.

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PHYSICAL MODELING OF PROCESSES AFFECTING THE DIFFUSION MECHANISM DURING CORROSION

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ARTICLE INFO	ABSTRACT
Article history: Received:2025-10-08 Received in revised form:2025-10-21 Accepted:2025-11-24 Available online:2026-06-30 Keywords: Diffusion; Adsorption; Adhesion; Corrosion; Mathematical modeling. JEL Classification: L61, O32, C63	The article analyzes the mechanism of the diffusion process, which is one of the fundamental kinetic stages occurring during the corrosion of metals and alloys. In this research, the temperature dependence of the diffusion coefficient, point defects in the crystal structure, phase transitions, and the thermodynamics of grain boundaries were investigated; furthermore, the relationship between the diffusion coefficient and both crystal structure and thermodynamic factors was analyzed using mathematical models. Specifically, the influence of excitation processes occurring at the atomic and nuclear levels, magnetic properties, and adsorption-adhesion phenomena on diffusion kinetics was analyzed from a physico-mathematical perspective (through mathematical modeling). These results indicate that a comprehensive, multiscale analysis of the diffusion mechanism (at the atomic, crystalline and nuclear levels) enables a more accurate prediction of corrosion processes. Furthermore, the proposed physico-mathematical approach provides an effective theoretical framework for evaluating the integral impact of various external factors on diffusion kinetics.

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1. INTRODUCTION

One of the most pressing issues in modern materials science is to gain a profound understanding of the dynamics of corrosion processes and to manage them effectively. One of the primary kinetic stages determining the corrosion rate is the diffusion process, which facilitates mass transport. The parabolic nature of mass transport, crystal lattice defects, and interphase transitions are factors that account for the complexity of this process [3,5]. Recent studies indicate that the phenomenon of corrosion is closely linked not only to macroscopic changes but also to changes occurring at the microscopic and subatomic levels. The state of nucleons (neutrons and protons) in the nucleus, energy distribution, and temperature variations across discrete levels can act as stimulating factors for the diffusion mechanism within a corrosive environment [1,2].

The diffusion process can occur through different mechanisms in various directions, which directly affects the dynamics of corrosion. The article provides a mathematical description of the mechanism of diffusion variation in different directions and examines results obtained from the

computer modeling of phenomena occurring at the atomic and nuclear levels during the initial stage of the process. It has been determined that under constant potential conditions, intranuclear processes do not significantly influence diffusion due to the weakness of the interaction between neutrons and protons. However, during moments of excitation and throughout nuclear reactions, magnetism and rising temperatures intensify the diffusion mechanism. Excitation states arising against the background of a constant nucleon potential lead to the distribution of energy across discrete levels within the nucleus and variations in temperature, which directly influence diffusion kinetics.

2. PHYSICAL AND MATHEMATICAL ANALYSIS

The analysis of factors stimulating corrosion and physical modeling for their prevention are of particular importance. Accordingly, one of the factors affecting the mechanism of the corrosion process is the mechanism of diffusion. However, a more complex and intriguing issue concerns the factors that influence the diffusion mechanism itself. In this regard, it is essential to investigate the parabolic nature of the mechanism, the influence of crystal structure on the diffusion coefficient, temperature dependence, interdiffusion, and thermodynamic factors.

The variation in diffusion mechanisms across different directions impacts the overall corrosion mechanism [1,7]. The mathematical description of the variation in diffusion mechanisms across different directions is expressed by the following equations:

$$\frac{\partial C(r,\tau)}{\partial \tau} = \frac{1}{r^n} * \frac{\partial}{\partial r} * \left(r^n D \frac{\partial C(r,\tau)}{\partial r} \right) \quad (1)$$

$$\frac{\partial C(r,\tau)}{\partial \tau} = D * \frac{\partial^2 C(x,\tau)}{\partial x^2} \quad (2)$$

$$\frac{\partial C(x,y,z,\tau)}{\partial \tau} = D \nabla (\nabla C(x,y,z,\tau)) = D \Delta C(x,y,z,\tau) \quad (3)$$

$$\frac{\partial C(x,y,z,\tau)}{\partial \tau} = D \left(\frac{\partial^2 C(x,y,z,\tau)}{\partial x^2} + \frac{\partial^2 C(x,y,z,\tau)}{\partial y^2} + \frac{\partial^2 C(x,y,z,\tau)}{\partial z^2} \right) \quad (4)$$

$$\frac{\partial C(x,\tau)}{\partial \tau} + \frac{\partial j(x,\tau)}{\partial x} = 0 \quad (5)$$

$$\frac{\partial C(x,y,z,\tau)}{\partial \tau} + \text{div } j(x,y,z,\tau) = 0 \quad (6)$$

$$V = \frac{dq}{d\tau} = DS \frac{\partial C(x,\tau)}{\partial x} \quad (7)$$

$$j = -D \text{grad} C(x,y,z,\tau) = -D \nabla C(x,y,z,\tau) \quad (8)$$

$$\text{Grad} = \nabla = \bar{i} \frac{\partial}{\partial x} + \bar{j} \frac{\partial}{\partial y} + \bar{k} \frac{\partial}{\partial z} \quad (9)$$

$$j = \frac{dq}{Sd\tau} \quad (10)$$

$$j_x = -D \frac{\partial C(x,\tau)}{\partial x} \quad (11)$$

$$\frac{dC(x)}{dx} \approx \frac{C_2 - C_1}{\Delta x} = \frac{\Delta C}{\Delta x} = \frac{C\left(x + \frac{\Delta x}{2}\right) - C\left(x - \frac{\Delta x}{2}\right)}{\Delta x} \quad (12)$$

$$dq = -DS \frac{\partial C(x,\tau)}{\partial x} d\tau \quad (13)$$

It is evident from the mathematical descriptions that diffusion varies in different directions depending on various parameters. These variations are associated with point defects, inter-defect interactions, phase transitions, and the thermodynamics of grain boundaries [4,6]. Furthermore, adsorption and adhesion phenomena, as well as environmental factors, significantly influence the process.

The diffusion mechanism is influenced by intra-atomic energy and magnetic moments, electron-nuclear interactions, and the state of nucleons (neutrons and protons) [2,8]. Let us examine the mathematical description of the processes occurring within the nucleus. As a result of computer modeling, the energy distribution function of the processes occurring in the nucleus has been described by equation (14):

$$\rho F(E_x) = \frac{\sqrt{\pi}}{12a^{1/4}E_x^{5/4}} e^{2\sqrt{aE_x}} \quad (14)$$

From this, it is clear that under a constant potential, neutrons and protons generally do not interact with each other [2, 5]. Where E^x - represents the excitation energy, and a - is the coefficient characterizing the magnetic properties between neutrons and protons. Naturally, in this case, the processes occurring in the nucleus will not significantly affect diffusion:

$$\lambda = \exp \left[a_0 + \sum_{i=1}^5 a_i T_9^{\frac{2i-5}{3}} + a_6 \ln T_9 \right] \quad (15)$$

As can be seen, in this case, it fully characterizes the duration of nuclear reactions during moments of excitation within the nucleus. This moment, as observed, indicates the point at which magnetism and temperature increase. At this stage, the diffusion mechanism and the diffusion from the corrosion-inducing source will intensify:

$$N(A, Z) = \sum_i \int_0^T \lambda_i(t) \cdot \prod_j y_j(t) dt \quad (16)$$

This is the mathematical description of the time at which neutrons leave the nucleus during this moment of excitation:

$$\lambda(T) = \sqrt{\frac{8}{\pi M}} \frac{N_A}{(kT)^{3/2} G(T)} \int_0^\infty \sum_p \frac{(2I^p+1)}{(2I^0+1)} \sigma^p(E) E \exp\left(-\frac{E+E_x^p}{kT}\right) dE \quad (17)$$

$$G(T) = \sum_p \frac{(2I^p+1)}{(2I^0+1)} \exp\left(-\frac{E_x^p}{kT}\right) \quad (18)$$

It is evident from these mathematical descriptions that if static equilibrium is established within the nucleus, no changes occur in the concentration of particles, and the energy distribution among neutrons remains constant. It is clear that nuclear reactions vary under the influence of temperature. However, simultaneously, the temperature within the nucleus rises during moments of excitation, and the energy distribution also changes depending on the discrete levels of these excitation moments. It has been clarified that the emergence of excitation moments during a constant nucleon potential is related to magnetic properties. The variation of energy and temperature across discrete levels within the nucleus during these different excitation moments directly impacts the diffusion mechanism during corrosion. However, the optimal solution to the stated problem involves investigating the influence of changes occurring in particles - especially within crystal lattices - on the diffusion mechanism for each excitation state. During these moments of excitation, the diffusion coefficient, as well as the thermodynamics, kinetics, and other specific mechanisms of diffusion, undergo a complete transformation.

The rise in temperature and changes in magnetism during excitation moments arising from nuclear reactions increase the diffusivity of the corrosion source. The research demonstrates that the excitation states occurring during a constant nucleon potential are directly linked to magnetic properties. In these cases, structural changes within the crystal lattices entirely alter the diffusion coefficient, as well as the thermodynamics and kinetics of the process.

One of the primary processes affecting the diffusion mechanism during corrosion is the mechanism of adsorption. Naturally, for an optimal solution to the problem, let us examine the physical description of the process:

$$dF^\alpha = -S^\alpha dT - p dV^\alpha + \sum \mu_i dN_i^\alpha \quad (19)$$

$$dF^\beta = -S^\beta dT - p dV^\beta + \sum \mu_i dN_i^\beta \quad (20)$$

$$\gamma = \left(\frac{\partial F}{\partial A}\right)_{T,V,N_I} \quad (21)$$

$$dF = -S dT - p dV + \sum \mu_i dN_i + \left(\frac{\partial F}{\partial A}\right)_{T,V,N_I} dA \quad (22)$$

It is clear from this description that in a heterogeneous system during adsorption, the very low number of molecules preserves the property of additivity; thus, if the adsorption mechanism proceeds via a certain pathway in one part, it tends to follow that same pathway in other parts. However, this mechanism varies due to several factors. The primary reasons for this include factors such as the nature of adsorption dynamics, the thermal effect, adsorption forces, variations in potential energy during adsorption, processes occurring in adsorption layers under the influence of temperature, and differential changes depending on temperature.

Certainly, during subsequent analysis, the individual examination of each parameter paves the way for an optimal solution to the problem.

One of the main processes and mechanisms that affect corrosion is the adhesion process and the formation of an adhesion layer. As such, the adhesion mechanism inherently influences the process. Let us examine the general mechanism of adhesion:

$$W_{adhesion} = \sigma_{s.t.c.} (1 + \cos \phi) \quad (23)$$

Simultaneously, the formation of the adhesion layer exerts a significant influence on the corrosion process. From this mechanism, it is evident that the adhesion process depends on the surface tension coefficient of the liquid ($\sigma_{s.t.c.}$) and the contact angle of the solid surface ($\cos \phi$). However, an optimal solution for investigating the influence of the adhesion process on the diffusion mechanism can only be found by examining the factors that affect these specific parameters. Analyses indicate that the adhesion strength depends on the energy per unit surface area (E) and the number of active bonds (N):

$$W_a = EN \quad (24)$$

These parameters, in turn, are closely linked to the state of the crystal lattice and processes occurring at the atomic-nuclear level; they vary depending on the nature of the bond, the condition of the crystalline lattice, and surface processes, consequently altering the adhesion mechanism.

Variations within the general mechanism - namely diffusion, adsorption, and adhesion - subsequently affect the corrosion process.

3. CONCLUSION

1. Although intranuclear processes do not significantly affect diffusion under constant potential conditions, increased temperature and magnetism during moments of excitation substantially accelerate the diffusion mechanism.
2. The distribution of molecules and the preservation of the additivity property during adsorption alter the character of the diffusion layers.

3. Adhesion strength influences corrosion resistance depending on the energy per unit surface area and the number of bonds.
4. Complex modeling of diffusion, adsorption, and adhesion mechanisms facilitates the discovery of optimal technological solutions for the corrosion protection of metal structures.

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AUTHOR GUIDELINES

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